

Sheet (1) Set of Rational Numbers (Q)

- The set of counting numbers C = { 1, 2, 3, 4, ... }
- The set of natural numbers $N = \{ 0, 1, 2, 3, 4, ... \}$
- The set of integer numbers $Z = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \dots\}$
- The set of rational numbers: the set contains the numbers can be written as a fraction and its denoted by Q, this fraction whose numerator is an integer and whose denominator is an integer except zero because (division by zero meaningless)

$$Q = \left\{ x : x = \frac{a}{b}, a \in z, b \in z, b \neq 0 \right\}$$

[1] Show which of the following number is rational and which is irrational:

(a) $\frac{2}{3}$

(b) zero

(c) 6.5

(d) - 1.8

(e) $5\frac{1}{6}$

(f) $\frac{-5}{3}$

 $(g) \quad \frac{-2}{3}$

(h) $\frac{2-2}{3}$

(i) $\frac{4}{5-5}$

(j) 3^2

(k) (-4)^{zero}

(l) 13%

[2] Show which of the following numbers is integer:

(a) $\frac{15}{5}$

(b) $\frac{4}{8}$

(c) $\frac{-35}{7}$

(d) $-\frac{14}{14}$

(e) $-\frac{24}{5}$

(f) $\frac{0}{5}$

(g) $3\frac{1}{4}$

(h) $\frac{3-3}{5}$

[3] Solve the following equations:

(1) $2 \times = 0$

(2) $4 \times = 0$

(3) x - 3 = 0

(4) x + 3 = 0

(5) 4 - x = 0

(6) x - 4 = 0



[4] Complete:

- (1) If $\frac{5}{a}$ is a rational number, then $a \neq \dots$
- (2) The number $\frac{3}{x-2}$ is a rational number if $x \neq \dots$
- (3) The number $\frac{x+7}{x-3} \in Q$ if $x \neq \dots$
- (4) The number $\frac{x+7}{x-3} \notin Q$ if $x = \dots$
- (5) The number $\frac{2}{5x}$ is a rational number if $x \neq \dots$
- (6) The rational number $\frac{4-x}{x-9} = 0$ if $x = \dots$
- (7) The rational number $\frac{x+5}{x-9} = 0$ if $x = \dots$

[5] Complete the following table:

The number	$\frac{5}{x-3}$	$\frac{3}{4-x}$	7 8x	6x x
Expresses a rational no. if $x \neq$				



[6] Complete the following table:

The rational number	$\frac{x-2}{x-1}$	$\frac{6-x}{x-4}$	$\frac{2x}{x+5}$	$\frac{2x-4}{x+3}$
Equals to zero if $x = \dots$				

[7] Write each rational number in the form $\frac{a}{b}$:

(1) - 5

(2) zero

(3) 0.75

(4) - 0.01

(5) 5.4

(6) 30%

(7) 4.5%

 $(8) 8\frac{2}{3}$



[8] Which of the following numbers can be written as a terminating decimal?

(1) $\frac{7}{15}$

(2) $\frac{-7}{20}$

(3) $\frac{-8}{9}$

 $(4) \quad \left| -1\frac{2}{9} \right|$

 $(5) \frac{17}{6}$

(6) $\frac{5}{11}$

(7) $\frac{5}{8}$

(8) $\frac{13}{22}$

(9) $2\frac{2}{5}$

(10) $-1\frac{2}{3}$

[9] Write each rational number as a decimal and a percentage:

The number	The decimal form	The percentage form
$(1) \frac{1}{4}$		
(2) $2\frac{1}{2}$		
(3) $\frac{21}{1000}$		
$(4) \frac{1}{6}$		
(5) - 3 20		
(6) 7 ³ / ₁₆		



[10] Put each of the following numbers in the simplest form:

(1)
$$\frac{15}{35}$$

(2)
$$\frac{-24}{56}$$

$$(3) \frac{45}{60}$$

(4)
$$\frac{-132}{88}$$

$$(5) \frac{33}{55}$$

(6)
$$\frac{36}{48}$$

[11] Complete:

(1)
$$\frac{2}{3} = \frac{4}{...} = \frac{...}{12} = \frac{...}{...}$$

(2)
$$\frac{4}{5} = \frac{8}{...} = \frac{...}{20} = \frac{...}{...}$$

(3)
$$6 = \frac{12}{...} = \frac{...}{3} = \frac{...}{...}$$

(4)
$$5 = \frac{35}{...} = \frac{...}{4} = \frac{...}{...}$$

[12] Choose the correct answer:

- If $\frac{4}{5} = \frac{20}{x}$, then x =(1)
 - (a) 25
- (b) -25
- (c)5
- (d) 100
- The number $\frac{a-6}{a-4}$ is not rational number if $a = \dots$
 - (a) 6
- (b) 4
- (c)1
- (d) zero
- The rational number $\frac{a}{b}$ is an integer if (3)
 - (a) a < b

(b) a > b

(c) b is a divisor of a

(d) a is a divisor of b

- $0.\overset{\bullet}{\mathbf{5}}\overset{\bullet}{\mathbf{7}}=\ldots\ldots$ (4)
 - (a) $\frac{57}{100}$
- (b) $\frac{57}{99}$
- (c) $\frac{575}{1000}$
- (d) $\frac{19}{33}$

- $(5) \quad \left| -\frac{8}{25} \right| = \dots$
 - (a) $-\frac{8}{25}$
- (b) $-0.3\dot{2}$ (c) $-0.3\dot{2}$
- (d) 32%

- (6) 12% =
 - (a) 0.3
- (b) 1.2
- (c) $\frac{3}{25}$
- (d) 0.012
- The rational number $\frac{x}{-3}$ is negative if (7)
 - (a) x > zero
- (b) x < zero
- (c) $x \le zero$ (d) x = zero
- If $\frac{a}{b}$ is a rational number and a b = zero, then (8)
 - (a) a = 0, $b \neq zero$

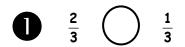
(b) $a \neq 0$, $b \neq zero$

(c) a = 0, b = zero

(d) $a \neq 0$, b = zero

Sheet (2) Comparing and Ordering Rational Numbers

[1] Complete using (<), (>) or (=):



$$2 \quad \frac{4}{5} \quad \bigcirc \quad \frac{3}{5}$$

$$7 - \frac{1}{2}$$
 zero

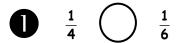
$$4 \quad \frac{1}{2} \quad \bigcirc \quad \frac{1}{4}$$

8
$$-\frac{3}{4}$$
 1 $\left|-\frac{3}{2}\right|$

$$\boxed{2} \quad \left| -\frac{3}{2} \right| \quad \boxed{2}$$



[2] Put the suitable sign using (<), (>) or (=):



$$\frac{9}{5}$$
 $\frac{9}{5}$ $\frac{12}{3}$

$$2 - \frac{5}{7} \bigcirc -\frac{3}{2}$$

2
$$-\frac{5}{7}$$
 $-\frac{3}{2}$ 4 $-3\frac{1}{2}$ $-\frac{20}{6}$ 6 1.6 $-\frac{8}{5}$



[3] Arrange the following rational numbers in a descending order:

$$\frac{3}{10}$$
 , $\frac{7}{30}$, $-\frac{1}{3}$, $-\frac{1}{5}$ and $\frac{4}{15}$





 $\frac{3}{4}$, $\frac{-5}{8}$, $-\frac{7}{12}$ and $\frac{2}{3}$



[5] Write a rational number in each of the following:

 $\frac{2}{5}$ < < $\frac{3}{5}$

- $\frac{2}{7}$ < < $-\frac{3}{14}$
- $2 \frac{2}{3} < \dots < -\frac{1}{3}$



[6] Write two rational numbers lying between:

- (1) $\frac{1}{2}$ and $\frac{4}{5}$
- (2) $-\frac{3}{4}$ and $-\frac{2}{3}$
- (3) 0.3 and $\frac{3}{5}$
- (4) $\frac{2}{5}$ and $\frac{3}{5}$
- (5) $\frac{1}{8}$ and $\frac{1}{4}$



[7] Write four rational numbers lying between:

- (1) $\frac{1}{2}$ and $\frac{11}{12}$
- (2) $-\frac{4}{9}$ and $-\frac{5}{6}$
- (3) Zero and 3



[8] If a = 3 and b = 5, which of the following numbers is rational and which is not?

- (1) $\frac{a}{2b}$
- (2) $\frac{b}{3-a}$

(3) $\frac{b-5}{a}$



[9] Identify and write four rational numbers between $\frac{3}{2}$ and $\frac{3}{4}$, such that one of them is an integer.

.....



Sheet (3) Adding and Subtracting Rational Numbers

Properties of the addition operation in Q:

(1) Closure property:

The sum of any two rational numbers is a rational number.

i.e.: Q is closed under addition operation.

(2) Commutative property:

If a and b are two rational numbers, then a + b = b + a

(3) Associative property:

If a, b and c are three rational numbers, then (a + b) + c = a + (b + c)

(4) Additive identity:

Zero is the additive identity (additive neutral element).

If a is a rational number, then

$$0 + a = a + 0 = a$$

(5) Additive inverse:

If a is a rational number, then

$$a + (-a) = zero$$

for example: $\frac{3}{5} + \left(\frac{-3}{5}\right) = zero$

Properties of the subtraction operation in Q:

Q is closed under subtraction operation, but the subtraction operation in Q is not commutative, not associative, has no identity element and has no inverse.



[1] Complete:

- (1) The additive identity element in Q is
- (2) The additive inverse of $\frac{3}{7}$ is

(3) The additive inverse of $-\frac{4}{9}$ is

(4) $\frac{-6}{-11}$ is the additive inverse of the number

(5) The additive inverse of $\left(\frac{2}{3}\right)^{zero}$ is

(6) The additive inverse of $\left(\frac{-2}{7}\right)^{zero}$ is

(7) The additive inverse of $\left|\frac{-4}{5}\right|$ is

(8) The additive inverse of zero is



[2] Complete:

(1) The remainder of subtracting $\frac{1}{5}$ from $\frac{6}{5}$ is

(2) The remainder of subtracting $\frac{1}{3}$ from $-\frac{4}{3}$ is

(3) The remainder of subtracting $-\frac{2}{3}$ from 0 is

(4)
$$\frac{1}{5}$$
 + = 0



[3] Find the result of each of the following in the simplest form:

(1)
$$\frac{3}{7} + \frac{2}{7} = \dots$$

$$(2) -\frac{3}{5} - \frac{9}{5} = \dots$$

(3)
$$\frac{7}{8} - \frac{3}{8} = \dots$$

$$(4) \quad \frac{5}{6} + \left(\frac{-4}{6}\right) = \ldots$$

$$(5) -\frac{2}{9} + \frac{2}{9} = \dots$$

$$(6) \quad \frac{5}{9} + \left| -\frac{4}{9} \right| = \dots$$

[4] Find the result of each of the following in the simplest form:

- $(1) \qquad -\frac{3}{10} + \left(-\frac{2}{5}\right) = \ldots$
- (2) $\frac{1}{4} + \frac{25}{8} = \dots$

(3) $\frac{-2}{5} - \frac{3}{15} = \dots$

(4) $\frac{1}{5} - \frac{2}{3} = \dots$

(5)
$$\frac{3}{7} - \left(-\frac{2}{5}\right) = \dots$$

(6)
$$\frac{19}{10} + \left(-\frac{39}{100}\right) = \dots$$

$$(7) -\frac{9}{12} + \frac{3}{16} = \dots$$



[5] Find the result of each of the following in the simplest form:

(1)
$$2\frac{2}{7} + 2\frac{3}{7} = \dots$$

(2)
$$9\frac{1}{5} - 7\frac{3}{5} = \dots$$

$$(3) \frac{1}{4} + 2\frac{3}{8} = \dots$$

$$(4) 6\frac{2}{3} - 3\frac{1}{6} = \dots$$

$$(5) -2\frac{1}{2} -12\frac{1}{16} = \dots$$

(6)
$$2\frac{3}{8} + \frac{1}{4} = \dots$$

(7)
$$\frac{2}{5}$$
 + 0.2 =

(8) 50% +
$$\frac{1}{4}$$
 =

(9)
$$\frac{2}{3}$$
 - 0.3 =



[6] Choose the correct answer:

- (1) $\frac{3}{4}$ + 50% =
 - (a) 75%
- (b) 150%
- (c) $\frac{5}{4}$
- (d) $\frac{3}{2}$

- (2) Subtracting $\frac{1}{5}$ from $\frac{6}{5}$ gives
 - (a) 1
- (b) -1
- (c) $\frac{-3}{5}$
- (d) $\frac{7}{5}$

- (3) Subtracting $\frac{1}{3}$ from $\frac{-4}{3}$ gives
 - (a) -1
- (b) 1
- (c) $\frac{-5}{3}$
- (d) $\frac{5}{3}$

- (4) Subtracting $\frac{1}{7}$ from zero gives
 - (a) zero
- (b) $\frac{1}{7}$
- (c) $\frac{-1}{7}$
- (d) $\frac{6}{7}$

- (5) Subtracting $\frac{-3}{2}$ from zero gives
 - (a) zero
- (b) $\frac{3}{2}$
- (c) $\frac{-3}{2}$
- (d) 1

- (6) $-\frac{1}{2} = -1$
 - (a) $1\frac{1}{2}$ (b) $\frac{1}{2}$
- (c) $-\frac{1}{2}$ (d) $-1\frac{1}{2}$

- $(7) \quad \frac{3}{5} + \dots = zero$
 - (a) $\frac{3}{5}$
- (b) $\frac{-3}{5}$
- (c) 1
- (d) zero

[5] Using the properties in Q, find out the result of each of the following in the simplest form:

(1) $\frac{1}{4} + \frac{1}{2} + \frac{3}{4}$

(2)	2	3	5	1
(2)	7	4	77	4

(4) $\frac{5}{8} + \frac{1}{3} + \frac{3}{8} + \left(\frac{-1}{3}\right)$

(3) $\frac{2}{13} + \frac{1}{5} + \frac{11}{13} + \left(\frac{-6}{5}\right)$

......

(5) $\frac{-3}{4} + \left(\frac{-3}{5}\right) + \left(-2\frac{1}{4}\right) + \frac{3}{5}$ (6) $\left|-\frac{1}{2}\right| + \left(-\frac{2}{4}\right) + \frac{6}{4} + \frac{1}{2}$

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[6] If $x = \frac{2}{3}$, $y = -\frac{1}{2}$ and $z = \frac{1}{6}$ find in the simplest form the numerical value of each of the following:

(1)
$$y + z$$

(2)
$$(x-y)-z$$



Sheet (4) Multiplying and Dividing Rational Numbers

Properties of the Multiplication operation in Q:

(1) Closure property:

The product of any two rational numbers is a rational number. i.e.: Q is closed under multiplication operation.

(2) Commutative property:

If a and b are two rational numbers, then $a \times b = b \times a$

(3) Associative property:

If a, b and c are three rational numbers, then $(a \times b) \times c = a \times (b \times c)$

(4) Multiplicative identity:

One is the multiplicative identity (multiplicative neutral element). If a is a rational number, then $1 \times a = a \times 1 = a$

(5) Multiplicative inverse (reciprocal of the number):

For any rational number $\frac{a}{b}$ except zero there is a multiplicative inverse that is the number $\frac{b}{a}$, where: $\frac{a}{b} \times \frac{b}{a} = 1$

- Zero has no multiplicative inverse because $\frac{1}{zero}$ is undefined.
- Multiplying any rational number by zero equals to zero.
- (6) Distribution property:

If a, b and c are three rational numbers, then $a \times (b + c) = a \times b + a \times c$ $a \times (b - c) = a \times b - a \times c$

Properties of operations:

operation Property	Addition	Subtraction	Multiplication	Division
Closure	✓	✓	✓	×
Commutative	✓	*	✓	×
Associative	✓	*	✓	×
Identity element	√ (0)	*	√ (1)	×
Inverse	✓	*	√except (0)	×



[1] Complete:

- (1) The multiplicative identity element in Q is
- (2) The multiplicative inverse of $\frac{3}{7}$ is
- (3) The multiplicative inverse of $\frac{-2}{3}$ is
- (4) The multiplicative inverse of -6 is
- (5) The multiplicative inverse of $3\frac{1}{2}$ is
- (6) The multiplicative inverse of 0.5 is
- (7) The multiplicative inverse of 1 is
- (8) The multiplicative inverse of -1 is
- (9) The multiplicative inverse of $\left(-\frac{3}{5}\right)^{zero}$ is
- (10) The multiplicative inverse of $\left|-\frac{3}{5}\right|$ is
- (11) The rational number that has no multiplicative inverse is
- (12) The rational number $\frac{a-1}{5}$ has a multiplicative inverse if $a \neq \dots$



[2] Put (\checkmark) for the correct statement and (*) for the incorrect one:

- (1) Every rational number has a multiplicative inverse. ()
- (2) The multiplicative inverse of a rational number is an integer. ()
- (3) The multiplicative inverse of the number $\frac{0}{7}$ is $\frac{7}{0}$.
- (4) The multiplicative inverse of the number $2\frac{1}{5}$ is $5\frac{1}{4}$.
- (5) The multiplicative inverse of the number $\left(\frac{2}{7} + \frac{3}{5}\right)$ is $\frac{35}{31}$.



[3] Complete:

The number	The additive inverse	The multiplicative inverse
3 7		
$\frac{-4}{9}$		
-6		
0.5		
$3\frac{1}{2}$		
$\left(\frac{-3}{8}\right)^{zero}$		
$\left -\frac{3}{7}\right $		
1		
-1		
0		
<u>1</u> 5		

[4] Complete:

(1)
$$\frac{3}{2} \times \left(\frac{-4}{5}\right) = \frac{-4}{5} \times \dots$$
 property

(2)
$$\frac{2}{3} \times \frac{3}{2} = \dots$$
 property

$$(3) \quad 7 \times \frac{\cdots}{7} = 1 \qquad \qquad \text{mnnn} \quad \text{property}$$

(4)
$$-\frac{4}{5} \times \dots = -\frac{4}{5}$$
 property

(5)
$$-\frac{4}{11} \times \dots = 1$$
 property

(6)
$$2\frac{3}{5} \times \dots = 1$$
 property

(8)
$$4 \times = -5$$
 property

(9)
$$\frac{2}{3}\left(2+\frac{1}{2}\right) = \frac{2}{3} \times 2 + ... \times ...$$
 property

(10)
$$\frac{3}{9} = \frac{2}{3} \times \frac{...}{8}$$

(11) If
$$\frac{x}{y} = \frac{2}{3}$$
 then, $\frac{3x}{2y} = \dots$

(12) If
$$\frac{a}{b} = 70$$
 then $\frac{a}{2b} =$



[5] Find out the result of each of the following in the simplest form:

$$(1) \quad \frac{3}{5} \times \frac{2}{7} = \dots$$

(2)
$$\frac{-1}{2} \times \frac{2}{3} = \dots$$

$$(3) \quad -\frac{3}{8} \times \left(-\frac{5}{3}\right) = \ldots$$

$$(4) \quad \frac{2}{6} \times \left(-\frac{3}{4}\right) = \dots$$

$$(5) \quad \left(-\frac{2}{3}\right) \times \frac{5}{8} = \dots$$

$$(6) \quad \frac{4}{5} \times \left(-\frac{5}{7}\right) = \dots$$

$$(7) \quad \left| -\frac{3}{7} \right| \times \left(-\frac{4}{3} \right) = \dots$$

(8)
$$\frac{1}{2} \times |-12| = \dots$$

[6] Find out the result of each of the following in the simplest form:

(1)
$$\frac{4}{5} \div \frac{3}{7} = \dots$$

(2)
$$-\frac{1}{6} \div \frac{5}{2} = \dots$$

(3)
$$\frac{-4}{11} \div \left(\frac{-4}{11}\right) = \dots$$

(4)
$$\frac{5}{27} \div \frac{1}{9} = \dots$$

$$(5) \quad \frac{5}{6} \div \left(\frac{-15}{2}\right) = \dots$$

(6)
$$\frac{-5}{8} \div \frac{5}{8} = \dots$$

(7) zero
$$\div \frac{3}{5} = \dots$$

(8)
$$1 \div \frac{7}{5} = \dots$$



[7] Find out the result of each of the following in the simplest form:

(1)
$$3\frac{1}{2} \times (-4) = \dots$$

$$(2) 1\frac{1}{2} \times \left(\frac{-3}{2}\right) = \dots$$

(3)
$$\left(-4\frac{2}{7}\right) \times \left(-5\frac{1}{6}\right) = \dots$$

$$(4) \quad 3\frac{1}{8} \times \left(-4\frac{1}{5}\right) = \ldots$$

$$(5) \quad \left(-1\frac{1}{2}\right) \times \left|-\frac{5}{3}\right| = \dots$$

(6)
$$0.6 \times 1\frac{1}{3} = \dots$$



[8] Find out the result of each of the following in the simplest form:

(1)
$$-2\frac{1}{5} \div \frac{11}{5} = \dots$$

(2)
$$-7\frac{5}{6} \div \frac{47}{100} = \dots$$

(3)
$$-4\frac{2}{7} \div 1\frac{1}{14} = \dots$$

(4)
$$-4\frac{1}{3} \div \left(-3\frac{1}{4}\right) = \dots$$

(5)
$$-2\frac{3}{4} \div \left(-3\frac{1}{8}\right) = \dots$$

(6)
$$6\frac{1}{4} \div (-15) = \dots$$

[9] Using the distribution property, find out the result of each of the following in the simplest form:

(1)
$$\frac{5}{12} \times 3 + \frac{5}{12} \times 9$$

(2)
$$\frac{4}{9} \times 11 + \frac{4}{9} \times 16$$

(3)
$$\frac{6}{37} \times 7 + \frac{6}{37} \times 5 + \frac{6}{37} \times (-11)$$

(4)
$$\frac{7}{12} \times 5 + \frac{7}{12} \times 9 - \frac{7}{12} \times 2$$

(5)
$$\frac{7}{13} \times 6 + \frac{7}{13} \times 8 - \frac{7}{13}$$

(6)
$$\left(\frac{-3}{7}\right) \times 8 + 5 \times \left(\frac{-3}{7}\right) + \left(\frac{-3}{7}\right)$$



[10] Find the result in the simplest form:

(1)
$$\left(\frac{3}{8} + \frac{5}{8}\right) \div \frac{5}{8} = \dots$$

(2)
$$\frac{3}{4} \times \left(\frac{1}{2} - \frac{1}{3}\right) = \dots$$

(3)
$$\left(\frac{-18}{5} \div \frac{9}{35}\right) \times \left(\frac{-3}{7}\right) = \dots$$
 (4) $-4\frac{1}{3} \div \left(-3\frac{1}{4}\right) = \dots$

(4)
$$-4\frac{1}{3} \div \left(-3\frac{1}{4}\right) = \dots$$

(5)
$$\left[\frac{-12}{25} \times \left(-\frac{5}{7}\right)\right] \div \left(\frac{-9}{14}\right) = \dots$$
 (6) $\left[\left(-1\frac{2}{3}\right) \times 4\frac{2}{3}\right] \div 6\frac{1}{9} = \dots$

(6)
$$\left[\left(-1\frac{2}{3}\right) \times 4\frac{2}{3}\right] \div 6\frac{1}{9} = \ldots$$



[11] Find the value of (n) in each of the following:

$$(1) \qquad \frac{-7}{3} \times \frac{-3}{7} = n$$

$$(2) \qquad n \times \frac{17}{3} = 1$$

- $(3) \quad \frac{-7}{3} \times n = 0$
- $(4) \qquad \frac{5}{7} \times n = \frac{5}{7}$
- (5) $n \times \left[\frac{1}{2} + \left(\frac{-3}{5}\right)\right] = n \times \frac{1}{2} + 5 \times \left(\frac{-3}{5}\right)$



[12] If a=2, $b=\frac{1}{2}$ and $c=\frac{3}{2}$, find in the simplest form the value of: $(a-b)\div c$



- [13] If $x = \frac{1}{3}$, $y = \frac{3}{4}$ and z = -3, find in the simplest form the numerical value of each of the following:
 - (1) xyz.....
 - (2) xy+zy.....



[14] If $x = \frac{3}{4}$ and $y = \frac{-5}{3}$, find in the simplest form the value of the expression:

 $\frac{x-y}{x+y}$

Sheet (5) Applications on Rational Numbers

- The distance between two numbers 2 and 5 is: |2-5| = |5-2| = 3 length units
- The distance between two numbers -2 and 3 is: |-2-3| = |3+2| = 5 length units
- From the side of the smallest number: s + f(g s)
- From the side of the greatest number: g f(g s)



Ex (1): Find the rational number lying at the middle of the way between 3 and 7.

The number =
$$s + f(g - s) = 3 + \frac{1}{2}(7 - 3) = 5$$

Or

The number =
$$g - f(g - s) = 7 - \frac{1}{2}(7 - 3) = 5$$



Ex (2): Find the rational number lying at the half-way between $\frac{2}{5}$ and $\frac{3}{7}$.

The number =
$$s + f(g - s) = \frac{2}{5} + \frac{1}{2} \left(\frac{3}{7} - \frac{2}{5} \right) = \frac{29}{70}$$



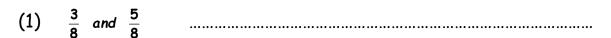
Ex (3): Find the rational number lying at one third of the way between 2 and 8.

From the side of the smaller number = $s + f(g - s) = 2 + \frac{1}{3}(8 - 2) = 4$

From the side of the greatest number = $g - f(g - s) = 8 - \frac{1}{3}(8 - 2) = 6$



[1] Find the rational number in the middle of the way (half-way) between:



(3)
$$\frac{1}{2}$$
 and $\frac{7}{8}$

(4)
$$\frac{-11}{4}$$
 and $\frac{-13}{35}$



[2] Find the rational number lying at:

(1) One fourth of the way between $\frac{5}{7}$ and $\frac{-3}{7}$ from the side of the smaller number.

.....

(2) One third of the way between $\frac{-3}{5}$ and $\frac{-4}{5}$ from the side of the greater number.

(3) One third of the way between $\frac{4}{7}$ and $1\frac{3}{4}$ from the side of the smaller number.

(4) One fifth of the way between $\frac{-2}{3}$ and $\frac{-3}{5}$ from the side of the

smaller number.



[3] Choose the correct answer:

- (1) If $a \times \frac{b}{2} = \frac{a}{2}$, $a \neq 0$, then b =
- (b) 0
- (c) a
- (d) 1 (e) -a
- (2) If $\frac{x}{3} 4 = 6$, then $\frac{x}{3} + \frac{2}{3} = \dots$
- (a) 1 (b) x (c) $\frac{32}{3}$
- (d) 10
- (e) $\frac{2x}{9}$

- (3) If $\frac{x}{y} = 1$, then 2x 2y =
 - (a) 4 (b) 2 (c) 1

- (d) 0
- (e) $\frac{1}{2}$

- (4) If $x + \frac{2}{x} = 5 + \frac{2}{5}$, then x =

 - (a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) 1
- (d) $\frac{5}{2}$
- (e)5

- (5) If 5a = 45 and ba = 1, then $b = \dots$
- (b) $\frac{1}{9}$ (c) $\frac{1}{5}$
- (d) 5
- (e) 9

- (6) The number $\frac{x-3}{x-5} \in Q$ if $x \neq \dots$
 - (a) 3
- (b) -3
- (c) 5
- (d) -5
- (e) 15



[4] Find three rational numbers lying between $\frac{3}{2}$ and $\frac{3}{4}$, such that one of them is an integer.

The perimeter and the area of some shapes

[1] The square:

$$P = S \times 4 = 4 S$$
 (coeff. = 4 and degree = 1st)

$$\Rightarrow$$
 A = S × S = S² (coeff. = 1 and degree = 2nd)

[2] Rectangle:

$$P = (\ell + \omega) \times 2 = 2(\ell + \omega)$$

$$A = \ell \times \omega = \ell \omega$$
 (coeff. = 1 and degree = 2^{nd})

[3] Parallelogram:

$$P = (x+y) \times 2 = 2(x+y)$$

[4] Rhombus:

$$\mathcal{F} A = S \times h = S h$$
 or $A = \frac{1}{2} \times d_1 \times d_2$

[5] Triangle:

P = the sum of all side lengths

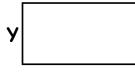
Perimeter of equilateral triangle = 3 S

Figure 1 If we denote one pound by x, if we have 3 pounds $x + x + x = 3 \times (coeff. = 3 \text{ and degree} = 1^{st})$

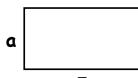
- The algebraic term is formed from the product of two or more factors.
- The degree of the algebraic term is the sum of the indices of the algebraic factors in this term.
- Any number is an algebraic term of zero degree.
- The algebraic term has no algebraic factors is called the absolute term.
- The algebraic expression consists of an algebraic term (monomial) or more.
- The degree of the algebraic expression is the highest degree of its terms.



[1] Write the algebraic term that represent the area of each shape:



X



5



[2] Complete the table:

Algebraic term	2 a b²	7 a b ³ c	-8 x² b	3	(-2) ³	$\frac{1}{2}x^3yz^2$
Coefficient						
Degree						

[3] Complete the table:

The Algebraic expression	No. of terms	Name	Degree
-3 α ⁵ b			
$3x^2 + y$			
$5x^3 - 7x + 4$			
$2a^2 b + 3a b^2 - a^2 b^2$			
$x^2 y^2 - 3x y^4$			
$a^2 b - 3a b^3 + 2a^3 b^2 + b^4$			



[4] Complete:

(1) The coefficient of algebraic term 3 x^2 y is and its degree is

(2) The coefficient of algebraic term $\frac{1}{2}x^3yz^2$ is and its degree is

(3) The degree of the absolute term in an algebraic expression is

(4) The algebraic expression $5x^2 + 3$ is of the degree.



[5] Choose the correct answer:

(2) The coefficient of the algebraic term $3xy^3z^4$ is (a) 2 (b) 3 (c) 6 (d) 7

The degree of the algebraic expression $3x^2 + 3x^4$ equals to the (3) degree of the algebraic expression

(a) $5xy+3y^2z$

(b) $2x^2y^2 + 3x^2y$ (c) $2xy + 3x^4z$ (d) $5a^2b + 4ab^2$

The number of terms of the algebraic expression $3x^2+5xy+6$ is ... (4)

(a) 1

(b) 2

(c)3

(d)4

The operation is unclosed in the set of rational numbers. (5)

(a) addition

(b) subtraction (c) division

(d) multiplication

If the degree of the algebraic term $2a^3b^n$ is ninth, then $n = \dots$ (6)

(a) 8

(b) 6

(c) 2

The algebraic term $b^3 = \dots$ (7)

(a)
$$3 b \times b$$

(b)
$$b + b + b$$

(a)
$$3 b \times b$$
 (b) $b + b + b$ (c) $b \times b \times b$ (d) $3 \times b$

(d)
$$3 \times b$$



[6] Arrange the terms of the following algebraic expressions according to the descending order of the indices of a:

(1) $5a + a^2 - 7 + a^3$

 $2 a^2 b^2 + 5 b a^3 - 3 b^3 a$ (2)



[7] Arrange the terms of the following algebraic expressions according to the ascending order of the indices of x:

(1) $5x + x^2 - 7 + x^3$



.....

(2) $2 x^2 y^2 + 5 y x^3 - 3 y^3 x$



Sheet (7) Like Algebraic Terms

The algebraic terms are said to be like if they having the same symbols and the same degree. Such as:

Like terms	Unlike terms
a 2a, a and -5a.	$\mathcal{F} 2x$, $-3x^2$ and $7x^3$
a 2 x^2y , 4 yx^2 and $-\frac{1}{2}x^2y$	$\mathcal{F} 4x^2$, $5xy$ and y^2

[1] Put (\checkmark) for the correct statement and (*) for the incorrect one:

- (1) The two algebraic terms x^2 and 2x are like terms. ()
- (2) The two algebraic terms 3 a b^2 and a b^2 are like terms. ()
- (3) The two algebraic terms $7x^2$ and $2x^7$ are like terms. ()
- (4) The two algebraic terms 3 a^2 b^3 and -2 b^3 a^2 are like terms. ()
- (5) $2 a + 3 a = 5 a^2$
- (6) $7 x^2 2 x^2 = 5 x^2$
- $(7) 8 y^2 5 y = 3 y ()$
- (8) 3 a b 3 b a = zero ()

[2] Find the result of each of the following:

(1) $3 \times + \times = \dots$

(2) 7 y - y =

(3) $3 \times + 2 \times = \dots$

(4) 5 y - 3 y =

(5) $4z - 11z = \dots$

- (6) $-7 \times -2 \times = \dots$
- (7) $2a + 3a 4a = \dots$
- (8) $-3 a^2 + 5 a^2 = \dots$

(9) $\frac{5x}{4} + \frac{3x}{4} = \dots$

(10) $\frac{3x}{5} - \frac{x}{5} = \dots$

[3] Answer each of the following:

(1) Subtract y² from -3y²

(2) Subtract $-6x^2y$ from $9x^2y$

(3) What is the increase -2x of -5x?

(4) What is the increase 3a²b of a²b?

(5) What is the decrease -3ab of 2ab?

(6) What is the decrease $6x^2y$ of $-7x^2y$?



[4] Complete:

(1) The result of subtracting 3a from 7a is

(2) The result of subtracting $3x^2$ from $-5x^2$ is

(3) The result of subtracting 7y³ from zero is

(4) The result of subtracting -3a from 2a is

(5) 5a increases 3a by

(6) 7x increases -3x by

(7) 4x decreases 7x by

(8) 5x decreases 3x by

(9) 2x decreases 4x by while 2x increases 4x by

(10) + $2a^2 = 7a^2$

(11) $3x^2 - \dots = x^2$

(12) $2m^2 + \dots = zero$

(13) $5 a^2 b - \dots = 7 a^2 b$

(14) If 4x - y = 11 and y = 3x, then $x = \dots$

[5] If the sum of two terms is $12 \times^2 y$ one of them is $4 \times^2 y$. Find the other term.



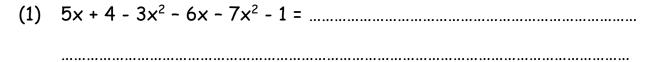
[6] Reduce to the simplest form:

(2)
$$2x - 4y - 9x - 3y = \dots$$

(3)
$$3x - 5y - x + 2y = \dots$$



[7] Reduce each of the following algebraic axpressions:



.....

(2)
$$6 x^2 y - 3 x y^2 + 2 x y^2 - 5 x^2 y + 2 x^2 y^2 = \dots$$

.....

(3)
$$a^2 + 4a - 5 + 3a^2 - 6a + 1 = \dots$$

.....

(4)
$$5 x^2 - 2 x + 8 - 7 x - 3 + x^2 = \dots$$



Sheet (8) Adding and Subtracting Expressions

[1] Find the sum of each of the following:

(1)	3 x -	2 v +	5 and x	(+ 2 y -	2
(-)		-	o una x	` - ,	_

.....

.....

(2) $3n^2 + 5n - 6$ and $-n^2 - 3n + 3$

.....

.....

(3) $3\ell - 4m + 5n$ and $4m - 5n - \ell$

(4) $3a^3-2a^2b+b^3$ and $a^3+4a^2b-b^3$

.....

[2] Find the sum:

(1) 3a + 2b - 5 , 2a - 7b + 4 , 5b - 4a + 3

.....

(2) 3x + 3y - z, 3x + 3z - 2y, x + 2y + z

.....

(3) $5x^2 - 3x + 9$, $x^2 + 2x - 5$, $x^2 - 3 - 6x$

.....

.....

(4) $3x - 4x^2 + 2$, $x^2 + x - 5$, $3 + 3x^2 - 4x$

.....

.....

.....



[3] Subtract:

(1) x - 2 from 2x - 5

(2) 2x + 6y - 7 from 2x - 5y + 2

[4] What is the increase of:

(1) 5a + 7b than 3a - 2b

(2) $x^2 - 5x - 1$ than $3x^2 + 2x - 3$

.....

[5] What is the decrease of:

(1) 2a + 3b than 5b - 3a

(2) $3y^2-2xy+x^2$ than $3x^2-5xy+y^2$

.....



[6] Subtract $x + x^2 - 5$ from $2x^2 + x - 3$, then find the numerical value of the result when x = 6



Sheet (9) Multiplying and Dividing Algebraic Terms

[1] Multiply:

(1)
$$5x \times 3y$$
 =

(2)
$$(-3a) \times 7c$$
 =

(3)
$$2x \times (-3x)$$
 =

$$(4) \quad \left(-8y^5\right) \times \left(-7y^4\right) \qquad = \dots$$

(5)
$$2xy \times (-3x^2)$$
 =

(6)
$$5x^3y^4 \times 2xy^2 = \dots$$

(7)
$$5ab^2 \times (-2a^2b)$$
 =

$$(8) \quad ab \times (-3a) \times (-2b) \qquad = \dots$$

(9)
$$2x^3 \times (-3x^2) \times (-5x^4) = \dots$$

(10)
$$(-2x) \times 4x$$
 =



[2] If the symbols represent non-zero integers, find the quotient of each of the following:

(1)
$$6a \div 2$$
 =

(2)
$$10c \div 2c$$
 =

(3)
$$12x \div (-x)$$
 =

(4)
$$(-14x^2) \div 7x$$
 =

(5)
$$\left(-25a^{6}\right) \div \left(-5a^{2}\right)$$
 =

(6)
$$24c^5 \div (-24c^5)$$
 =

(7)
$$9x^5y^4 \div 6x^3y = \dots$$

(8)
$$\left(-32a^3b^6\right) \div \left(-4a^3b^2\right) = \dots$$

(9)
$$8m^4n^3 \div (-4m n^2) = \dots$$



[3] Simplify:

(1)
$$\frac{2}{3}t^4 \times \frac{3}{2}t^4$$
 =

(2)
$$\frac{2}{7}a^2 \times 21a^5$$
 =

(3)
$$\frac{6x^4y^2}{7} \times \frac{28x y^3}{3} = \dots$$

(4)
$$3x^3 \times \frac{1}{6}x^2$$
 =



[4] Choose the correct answer:

(1)
$$3a^4b \times 5a^2b^2 \times 2a^3 = \dots$$

- (a) $60a^{11}b^3$ (b) $30a^{10}b^2$
- (c) $150a^{10}b^3$ (d) $30a^9b^3$

 $(2) \quad \left(-3x^2y\right)^2 \times 2xy = \dots$

(a)
$$-18x^5y^3$$
 (b) $18x^5y^3$ (c) $6x^3y^2$ (d) $9x^2y^2$

(b)
$$18x^5v^3$$

(c)
$$6x^3y^2$$

(d)
$$9x^2y^2$$

 $\left(-6x^3y^2\right) \div 3x^2y = \dots$ (3)

$$(a) - 2x^2y$$

$$(c) -2xv$$

(a)
$$-2x^2y$$
 (b) $2xy$ (c) $-2xy$ (d) $-2x^2y^2$

(4) If 2b cm is the edge length of a cube, then its volume = cm³

- (a) $4b^2$
- (b) $2b^3$
- (c) $4b^3$
- (d) $8b^3$

If the area of a rectangle is $24x^3$ cm² and its length is $8x^2$ cm, (5) then its width is

- (a) 3x
- (b) $3x^2$
- (c) 4x (d) $4x^5$



[5] Complete:

(1)
$$9a^5 = 3a \times$$

(2)
$$36a^5b^8 = 12a^3b^2 \times \dots$$

(3)
$$-4c^3d^3 = 2c d^2 \times$$

(4)
$$81l^4 \div \ldots = 27l^3$$

(5)
$$\dots \div 6a^2 = -4a^4$$

(6)
$$36a^7b^4 = \dots \times 9a^7b$$

Sheet (10) Multiplying a monomial by an algebraic expression

[1] Find the following products:

$$(1) \quad a(a+1) \qquad \qquad = \dots$$

(2)
$$a(a-2)$$
 =

(3)
$$3x(7y-4z)$$
 =

$$(4) -3(y+3) = \dots$$

(5)
$$-2c(7-3c)$$
 =

(6)
$$2x(3x^2 + 4y^2)$$
 =

(7)
$$-5x(2x + y - 3z)$$
 =

(8)
$$3xy(2x^2 - 5x^2y - 4y^2) = \dots$$

(9)
$$l m^2 (l^2 - 3m l - 4m^2) = \dots$$

(10)
$$\frac{1}{3}x^2(6x^2 - 9xy - 3y^2) = \dots$$

[2] Put in the simplest form:

(1) 3a(a-b) + 4a(2a+b) =

(2) 3a(4a-2)-4a(3a-2) = ...

.....



[3] Simplify 2a(3a-1)+3a(a+2), then find the numerical value of the result when a=1:

2a(3a-1)+3a(a+2) = ...

.....



Sheet (11) Multiplying a binomial by an algebraic expression

We have 3 ideas of the examples on this lesson

1st idea this is the general idea

[1] Find by direct products:

(1)
$$(x + 3)(x + 2)$$
 =

(2)
$$(x-3)(x-2)$$
 =

(3)
$$(x+2)(x-5)$$
 =

(4)
$$(y-4)(y+5)$$
 =

(5)
$$(x + 2)(x + 4)$$
 =

(6)
$$(y-5)(y+2)$$
 =

(7)
$$(5m-2)(6m+1)$$
 =

(8)
$$(4x + 1)(2x + 3)$$
 =

(9)
$$(3a + 2b)(2a - 5b)$$
 =

(10)
$$(b^2 - 4)(b^2 + 2)$$
 =

(11)
$$(x-y)(7y-x)$$
 =



2nd idea (special case of 1st idea)

[2] Find by inspection the expansion of each of the following:

(1)
$$(x + 2)^2$$
 =

(2)
$$(x + 3)^2$$
 =

(3)
$$(x + 1)^2$$
 =

(4)
$$(x-1)^2$$
 =

(5)
$$(2y + 3)^2 = \dots$$

(6)
$$(4m-7)^2 = \dots$$

(7)
$$(3x + y)^2 = \dots$$

(8)
$$(x - 3y)^2 = \dots$$

(9)
$$(2x + 3y)^2 = \dots$$

(10)
$$(-l-m)^2 = \dots$$

$$(11) (-4x-7)^2 = \dots$$



3rd idea special case of 1st idea

[3] Find by inspection the expansion of each of the following:

(1)
$$(x + 3)(x - 3)$$
 =

(2)
$$(x-4)(x+4)$$
 =

- (x-2)(x+2)(3) =
- (4m-7)(4m+7)(4) =
- $(6x + 2y)(6x 2y) = \dots$ (5)
- $(a^2+a)(a^2-a)$ (6) =
- $(3x^2 + 5y^2)(3x^2 5y^2) = \dots$
- (8) $\left(\frac{1}{2}x + \frac{1}{3}y\right)\left(\frac{1}{2}x \frac{1}{3}y\right) = \dots$



[4] Choose the correct answer:

- The middle term in the expansion of $(3x 1)^2$ is (1)
 - (a) 3x
- (b) -6x
- (c) 6x
- (d) $6x^2$
- The middle term in the expansion of $(2a + 3b)^2$ is (2)
 - (a) 12ab
- (b) -12ab
- (c) 6ab
- If $(2x + y)^2 = 4x^2 + k x y + y^2$, then $k = \dots$ (3)
 - (a) 2
- (b) 4
- (c) 8
- (d) 6
- If x = -1, then the numerical value of $(x + 1)^2$ is (4)
 - (a) zero
- (b) 1
- (c) 2
- (d)3
- If $x^2 = 16$, $y^2 = 9$ and xy = 12, then $(x y)^2 = \dots$ (5)
 - (a) 49
- (b) 165
- (c) -1
- (d) 1
- If $(x + y)^2 = 26$ and $x^2 + y^2 = 20$, then $x y = \dots$ (6)
 - (a) 3
- (b) 6
- (c)9

If x + y = 7, then the numerical value of $x^2 + 2xy + y^2 = ...$ (7)

- (a)7
- (b) 14
- (c)49

If x - y = 3 and x + y = 5, then $x^2 - y^2 = \dots$ (8)

- (a) 2
- (b) -2
- (c) 8

(9) If $x = \frac{4}{3}$, then $(x-2)(x+2) = \dots$

- (a) $\frac{4}{3}$ 2 (b) $\left(\frac{4}{3}\right)^2$ 2 (c) $\left(\frac{4}{3}\right)^2$ 4 (d) $\left(\frac{4}{3}\right)^2$ + 4

(10) If $(x-3)(x+3) = x^2 + k$, then $k = \dots$

- (a) 9
- (b) 6
- (c) -9
- (d) -6

If $(x-y)(2x+y) = 2x^2 + k x y - y^2$, then $|k| = \dots$

- (a) -1
- (b) 1
- (c)3

[5] Multiply, then find the numerical value of the expression when

x = 1 and y = -2: $(x-5y)(x+5y) = \dots$ (1)

(3x + y)(x + 3y) =(2)

(x + 4)(3x + 2) =(3)

[6] Reduce $(x-y)^2 + 2xy$, then find the numerical value of the result when x = -1 and y = -2:

.....



[7] Reduce $(2x-2)^2 + (x-2)(x+2)$, then find the numerical value of the result when x = -1:



[8] Simplify to the simplest form (2a-3)(2a+3)+7, then find the numerical value of the result when a=-1:

.....

Sheet (12)

Dividing an algebraic expression by a monomial

[1] If the symbols in the following expressions are non-zero numbers, find the quotient in each case:

(1)
$$5a - 10$$
 by 5 =

(2)
$$4a^2 + 6a$$
 by $2a$ =

(3)
$$12a^2b + 20a b^2$$
 by $4a b$ =

(4)
$$16a^3b^2 - 24a^2b^2$$
 by $4a^2b$ =

(5)
$$12x + 15y$$
 by -3 =

(6)
$$24x^3 - 18x^2$$
 by $-6x^2$ =

(7)
$$60x^6 - 48x^{10} - 12x^3$$
 by $-12x^3 = \dots$

(8)
$$32x^5 - 48x^3 + 72x^7$$
 by $-8x^3 = \dots$



[2] Find the quotient of each of the following:

$$(1) \quad \frac{26x^2 + 14x^4}{2x} \quad = \dots$$

$$(2) \quad \frac{18m^4 + 32m^2}{-2m^2} \quad = \dots$$

(3)
$$\frac{48x^3 - 80x^2}{8x^2} = \dots$$

$$(4) \quad \frac{9l^3m^4 - 18l \ m^2}{3l \ m^2} = \dots$$

[3] Choose the correct answer:

- $(x^2 + x) \div x = \dots, \quad x \neq 0$ (1)
 - (a) zero (b) x
- (c) 2x + 1 (d) x + 1

- $(15a + 5) \div 5 = \dots$ (2)
 - (a) 3a
- **(b)** 10a
- (c) 3a+1 (d) 4a

- $(4a^3-2a)\div(-2a)=\ldots, a\neq 0$ (3)

 - (a) $-2a^2$ (b) $-2a^2 + 1$ (c) $2a^2 + 1$ (d) -1

- $(15x^4 + 5x^3) \div 5x^3 = \dots$ (4)
 - (a) $3x^2 + x$ (b) $5x^2 + 1$ (c) 3x + 1 (d) $4x^4$

- (5) $(3x^2y) \div 3x \ y = x 2y$

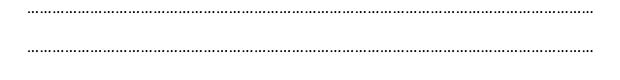
- (a) 6x (b) $6x y^2$ (c) $6y^2$ (d) $-6x y^2$
- If $(6x^2y^3 + k \ x \ y) \div 6x = x \ y^3 12y$, $x \neq 0$, then $|k| = \dots$
 - (a) -72 (b) -2 (c) 2
- (d) 72

Sheet (13)

Dividing an algebraic expression by another one

[1]	Find	the	quotient	of	each	of	the	following
-----	------	-----	----------	----	------	----	-----	-----------

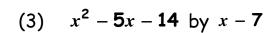
(1)	x^2	+	5 <i>x</i> +	6	by	x	+	2
-----	-------	---	---------------------	---	----	---	---	---



.....

(2) $y^2 - 9y + 20$ by y - 4

.....



(4) $2x^2 + 13x + 15$ by x + 5

.....

(5) $3x^2 + 2x - 8$ by 3x - 4

.....

.....

(6) $x^2 - 6 - x$ by x + 2

.....

[2] If the area of a rectangle is $(15x^2 + 11x - 14)$ cm² and its width is (3x - 2) cm. Calculate its length.

[3] If the area of a rectangle is $(2x^2 + 7x - 15)$ cm² and its length is (x + 5) cm. Find its width and calculate its perimeter when x = 3.

Sheet (14) Factorization by identifying the H.C.F.

1st idea we take the repeated number or repeated symbol out

[1] Factorize each of the following by identifying the H.C.F.:

(1)
$$5a + 5b$$
 =

(2)
$$3x - 3y$$
 =

(3)
$$7x y + 7y z = \dots$$

(4)
$$5a - 5b + 5c = \dots$$

(5)
$$3x(a+b) + 7(a+b) = \dots$$

(6)
$$a(a+3)+b(a+3) = \dots$$

(7)
$$(x + 4)x^2 + (x + 4)y^2 = \dots$$



2nd idea if one of the two terms divisible by the other

[2] Factorize each of the following by identifying the H.C.F.:

(1)
$$5y + 10$$
 =

(2)
$$8y^3 - 4x^2 = \dots$$

(3)
$$5a \ b - 15b \ c = \dots$$

(4)
$$3x^2 + 6x = \dots$$

(5)
$$35a + 10a^2$$
 =

(6)
$$4a b^2 - 7b^3 = \dots$$

(7)
$$35x^3y - 5xy^2 = \dots$$

(8)
$$15a^3b - 5a^2b^2$$
 =



[3] Factorize each of the following by identifying the H.C.F.:

(1)
$$6a^3 - 4a^2b^2$$
 =

(2)
$$6a + 8b - 10c$$
 =

(3)
$$x^3 + 2x^2 + 5x$$
 =

(4)
$$2x^2y + 6xy^2 - 2y$$
 =

(5)
$$9m^4n^2 - 6m^3n^3 + 12m^2n^4 = \dots$$

(6)
$$-2x^5 + 4x^2 - 6x + 2x^3 = \dots$$

(7)
$$18a^2b c - 6a b c + 30a b c^2$$
 =

(8)
$$15a^3b^4 + 6a^5b^3 - 3a^2b^2 = \dots$$

(9)
$$14a(x + y) - 21b(x + y)$$
 =

(10)
$$6a^2(x-1)-8a(x-1)$$
 =

(11)
$$3x^2(x-7) + 2x(x-7) + 5(x-7) = \dots$$

(12)
$$4m^2(2x + y) - 3m(2x + y) - 7(2x + y) = \dots$$



[4] Find the result by identifying the H.C.F.:

(1) 48 × 45 + 48 × 55

=

=

(2) $7 \times 123 + 7 \times 35 - 7 \times 18$

=

=

(3) $15 \times 17 + 15 \times 13 - 15 \times 30$

=

=

 $(4) (256)^2 - 256 \times 156$

=

=

=

(5) $6 \times (15)^2 + 18 \times 15 - 8 \times 15$

=

=

=

(6) $5 \times (48)^2 + 7 \times 48 + 53 \times 48$

=

=

=

[5] Complete:

(1)
$$6a^2 + 12a b = 3a(.... +)$$

(2)
$$a^2b + b^2a = \dots (a+b)$$

(3)
$$3(a-b)-4(b-a)=....(a-b)$$

(4)
$$x(a+1)-y(a+1)=(a+1)(....-...)$$

(5) If
$$a + b = 3$$
, then $5a + 5b =$

(6) If
$$7x - 7y = 21$$
, then $x - y =$

(7) If
$$2x + y = 7$$
 and $a + b = 3$, then $2x(a + b) + y(a + b) =$



[6] Choose the correct answer:

- $3x 9x^2 = \dots$ (1)
 - (a) 12x

- (b) -6x (c) $-6x^2$ (d) 3x(1-3x)
- (2) $7x^2 + 14y^2 = 7(\dots)$
- (a) $x^2 + y^2$ (b) $x^2 + 2y^2$ (c) $7x^2 + y^2$ (d) x + 2y
- (3) $4x^2y^2 2xy^2 + 4x^2y = \dots (2xy y + 2x)$
 - (a) 4x y (b) 2x y (c) 2x

- $(75)^2 + 75 \times 25 = \dots$ (4)
 - (a) 75
- (b) 750
- (c) 7500
- (d) 75000

- $8 + 8^2 = 8 \times \dots$ (5)
 - (a) 8
- (b) 9
- (c) 80
- (d) 90
- The H.C.F. of the expression $12x^3y^4 + 8x^2y^3$ is (6)
 - (a) $2x^2y^3$
- (b) $4x^2y^3$
- (c) $4x^3y^4$ (d) $12x^3y^4$

Sheet (15) The mode

[1] Complete:

- (1) The mode of a set of values is
- (2) The mode of the values 6, 5, 7, 6 is
- (3) The mode of the values 2, 3, 8, 2, 9 is
- (4) The mode of the values 3, 6, 10, 13, 19, 19, 21 is
- (5) The mode of the values 5, 33, 5, 33, 3, 5 is
- (6) The mode of the values 8, 11, 5, 8, 4, 5, 4, 11, 4 is
- (7) If the mode of the values 4, a, 5, 3, is 3 then a =
- (8) If the mode of the values $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{5}$, $\frac{1}{7}$ is $\frac{1}{x}$ then $x = \dots$
- (9) If the mode of the values 12, 7, x + 1, 7, 12 is 12 then $x = \dots$
- (10) If the mode of the values a+2, a+1, a+3, a+2 is 12 then a =



[2] The following frequency table represents the marks of 40 pupils in an examination:

The mark	15	16	17	18	19	20
No. of pupils (frequency)	4	5	8	12	7	4

Find the mode mark.



[3] The following frequency table shows the number of studying hours of 30 pupils in a week:

The number of studying hours	25	26	27	28	29	30
No. of pupils (frequency)	3	5	12	6	3	1

Find the mode number of studying hours.



[4] The following frequency table shows the maximum temperature degree registered in some arabic capitals:

Temperature degree	18	19	20	21	22	23
No. of captials (frequency)	3	2	4	6	2	1

- (1) Represent these data by bar charts graph.
- (2) Find the mode number of temperature degrees.



Sheet (16) The median

	[1]	Choose	the	correct	answer:
--	-----	--------	-----	---------	---------

1	11	The	median	٥f٠	1	Q	3	ic	
(1)	ıne	meaian	OT:	4.	Ö,	3	IS	

(a) 3

(b) 4

(c)5

(d) 8

(2) The median of: 6, 5, 9, 8 is

(a)5

(b) 6

(c)7

(d) 7.5

(3) The median of: 8, 17, 4, 6, 10 is

(a) 11

(b) 10

(c) 8

(d) 6

(4) The median of: 3, 7, 2, 9, 5, 11 is

(a)5

(b) 6

(c)7

(d) 12

(5) The median of: 25, 32, 28, 40, 50, 58, 50 is

(a) 40

(b) 45

(c)50

(d) 58

(6) The median of: 2, 5, 5, 6, 7, 9, 11, 14, 16, 21 is

(a)7

(b) 8

(c)9

(d) 16

(7) The order of the median of: 6, 2, 5, 4, 1 is

(a) 1st

(b) 2nd

(c) 3rd

(d) 4th

(8) If the order of the median of a number of oredered values is the third, then the number of these values is

(a) 3

(b) 4

(c)5

(d) 6

[2] Write these numbers in an ascending order, then find the median:

2.9, 2.3, 1.6, 9.1, 2.8, 0.7, 8.1, 7.3, 6.2, 5.3, 12.2, 4.3, 8.5



[3] Write these numbers in a descending order, then find the median:

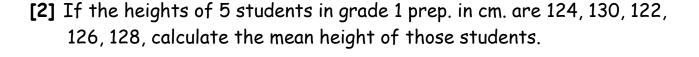
17.9, 7.4, 25.7, 8.9, 16.6, 3.8, 10.3, 32.3, 13.7, 0.5, 20.3, 16.3

.....

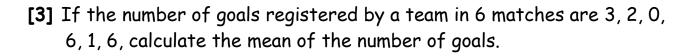
Sheet (17) The arithmatic mean

[1] Choose the correct answer from the given ones:

(1)	The mean of: 5, (a) 4	, 12, 6, 17 is (b) 5	(c) 6	(d) 10
(2)		5, 8, 9, 14, 28 is (b) 8	` ,	(d) 11
(3)	The mean of: 3, (a) 4	zero, 4, 6, 7 is . (b) 5	 (c) 6	(d) 7
(4)	The mean of: 2-(a) 1	-a, 4, 1, 5, 3+a is (b) 2	 (c) 3	(d) 15
(5)	The mean of: $x+$ (a) 3	y, 9-y, -x is (b) 9	(c) 2	(d) zero
(6)	The mean of: x , (a) x y	x-y, $y-x$ is (b) $\frac{y}{2}$	(c) $\frac{x}{2}$	(d) $\frac{x}{3}$
(7)	If the mean of: (a) 2	9, 4, 5, <i>x</i> is 5, tl (b) 3	nen x = (c) 4	(d) 5
(8)	If the mean of: (a) 29	3, 4, 8, a, a+2 is (b) 58		 (d) 17
(9)	If the mean of: (a) 18	<i>x</i> -1, <i>x</i> , <i>x</i> +1 is 6, 1 (b) 9	then <i>x</i> = (c) 15	(d) 6
(10)	marks is m	arks		the sum of their (4) 100
(11)		•		(d) 100 is 7 years old and esam is years
(12)	(a) 6	(b) 7	(c) 8	(d) 15
(12)		t side lengths (ne triangle is	•	s 8 cm, then the
	(a) 8	(b) 18	(c) 24	(d) 15









[4] This table shows the number of hours that the two athletes trained in each month of the year:

Gamal	75	72	68	46	57	66	63	70	58	30	48	53
Ali	62	64	54	52	63	68	56	65	70	50	49	57

- (1) Calculate the mean of the number of training hours of Gamal.
- (2) Calculate the mean of the number of training hours of Ali.







[1] The line segment:

It is the set of points between two distinct points and denoted by

$$\overline{AB}$$
 or \overline{BA} A C B D $AB = 6 \, cm$, $C \in \overline{AB}$, $D \notin \overline{AB}$

[2] The ray:

It is a line segment extended from only one of its terminals infintly and denoted by \overrightarrow{AB} $\stackrel{\bullet}{E}$ $\stackrel{\bullet}{A}$ $\stackrel{\bullet}{B}$ $\stackrel{\bullet}{C}$ $\stackrel{\bullet}{D}$ $\stackrel{\bullet}{D}$ $\stackrel{\bullet}{C}$ \stackrel

[3] The straight line:

It is a line segment extended from its two terminals infinitely and denoted by \overrightarrow{AB} or \overrightarrow{BA} $\stackrel{\bullet}{E}$ $\stackrel{\bullet}{A}$ $\stackrel{\bullet}{B}$ $\stackrel{\bullet}{C}$ $\stackrel{\bullet}{D}$ $\stackrel{\bullet}{D}$ $\stackrel{\bullet}{C}$ $\stackrel{\bullet}{C}$ $\stackrel{\bullet}{D}$

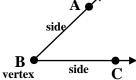
[4] The plane:

A plane is a flat unbounded surface and it is extended without limit in all directions.

[5] The angle:

It is the union of two rays having the same starting point (vertex of the angle) the two rays are called two sides of the angle.

$$\angle ABC$$
 , $\angle CBA$ or $\angle B$ $\overrightarrow{BC} \cup \overrightarrow{BA} = \angle ABC$



Types of angles:

- (1) Zero angle: Its measure = 0°.
- (2) Acute angle: 0° < its measure < 90°.
- (3) Right angle: Its measure = 90°.
- (4) Obtuse angle: 90° < its measure < 180°.
- (5) Straight angle: Its measure = 180°. ← →
- (6) Reflex angle: 180° < its measure < 360°.



[1] Complete the following table:

$m(\angle B)$	50°		105°		179°		115° 46′	
$m(reflex \angle B)$		330°		237°		350°	•••••	200° 19′ 30″



[2] Mention the type of the angle whose measure is as follows:

(1) 57°

(2) 117°

(3) 90°

(4) 200°

(5) 180°

(6) 43^{1°}

- (7) 179° 62'
- (8) $90\frac{2^{\circ}}{5}$



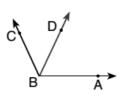
[3] From the opposite figure, complete using (\in) , $(\not\in)$, (\subset) or $(\not\subset)$:

- (1) $A \qquad \dots \qquad \overrightarrow{DC}$
- (2) $D \qquad \dots \qquad \overline{AC}$
- (3) $C \qquad \dots \qquad \overrightarrow{AB}$
- (4) A ∠ EBC
- (5) \overline{DC} \overrightarrow{AB}
- (6) \overline{BC} \overline{BA}
- (7) \overrightarrow{BA} \overrightarrow{DC}
- (8) \overline{AC} \overline{AD}

Sheet (2) Some Relations Between Angles Some Relations Between Angles

Adjacent angles

Two angles are said to be adjacent if they have a common vertex, a common side and the other two sides are on opposite sides of the common side.



∠ABD, ∠DBC are adjacent



Complementary angles

Two angles are said to be complementary if their sum is 90°.

And the two outer sides are perpendicular

[1] Write the measure of the angle which complements each of the angles whose measures are as follow:

(1) 30°

(2) 60°

(3) 48°

(4) 0°

(5) 90°

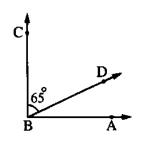
(6) 22^{1°}

(7) 25° 30′

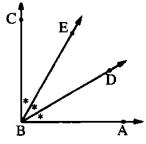
(8) $53\frac{1}{4}^{\circ}$

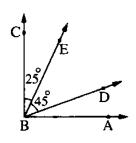


[2] In each of the following figures $\overrightarrow{BA} \perp \overrightarrow{BC}$, Complete:

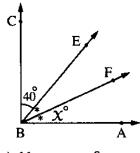


(1) m (∠ ABD) = ······°





(3) m (\angle ABD) = ·······°

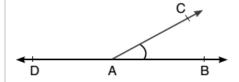


(4) X = ······°

Supplementary angles

Two angles are said to be supplementary if their sum is 180°.

Two adjacent angles formed by a straight line and a ray with a starting point on this straight line are supplementary



 $m (\angle BAC) + m (\angle CAD) = 180^{\circ}$

- [3] Write the measure of the angle which supplements each of the angles whose measures are as follow:
 - (1) 20°

 $(2) 90^{\circ}$

(3) 152°

(4) 0°

(5) 180°

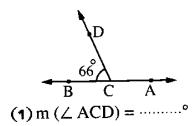
(6) $92\frac{1}{2}^{\circ}$

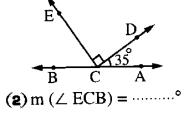
(7) 141° 24′

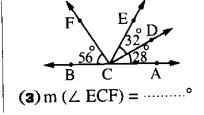
(8) 10°

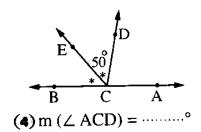


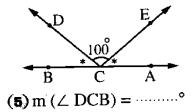
[4] In each of the following figures $C \in \overrightarrow{AB}$, Complete:

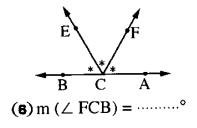


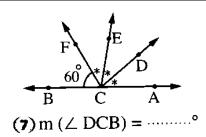


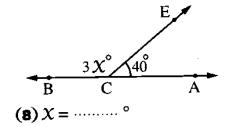








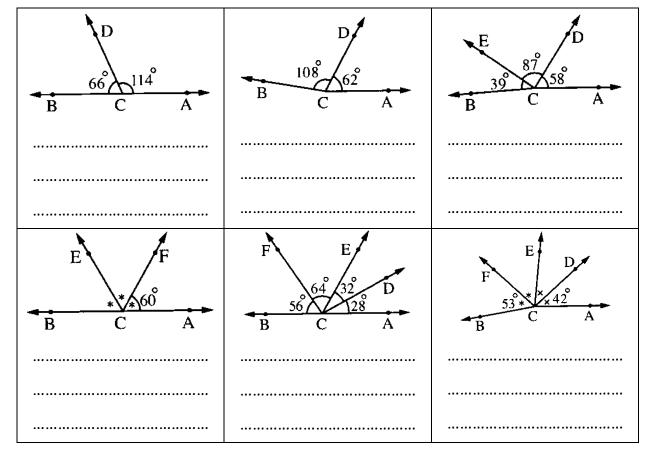






If two adjacent angles are supplementary then their outer sides are on the same straight line.

[5] In each of the following figures, state if \overrightarrow{CA} and \overrightarrow{CB} are on the same straight line or not, and why?



[6] Complete the following:

	omplete the following:
(1)	The angle is ·······
(2)	The measure of the straight angle = ········ o and the measure of zero angle is ········ o
(3)	The measure of the right angle = ········ °
(4)	The acute angle is the angle whose measure is less than and more than
(5)	The two complement angles are the two angles whose sum of their measures is
(6)	The two supplement angles are the two angles whose sum of their measures is
(7)	The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are
(8)	If the two outer sides of two adjacent angles are perpendicular, then these two adjacent angles are
(9)	If the two outer sides of two adjacent angles are on the same straight line, then these two adjacent angles are
(10)	If the two adjacent angles are supplementary, then their outer sides are
(11)	If the sum of measures of two adjacent angles does not equal 180°, then their outer sides are
(12)	The measure of the angle which is equivalent to two right angles = and it is called angle.
(13)	The angle whose measure is 50° complements an angle of measures and supplements the angle whose measure is
(14)	The angle whose measure complements the angle whose measure is 30° and supplements the angle whose measure is
(15)	The angle whose measure complements the angle whose measure is and supplements the angle whose measure is 150°
(16)	The acute angle complements angle and supplements angle.
(17)	Zero angle is complemented by angle and is supplemented by angle.
(18)	The right angle is complemented by angle and is supplemented by angle.



[7] Choose the correct answer:

- (1) The obtuse angle supplements angle.
 - (a) obtuse
- (b) right
- (c) acute
- (d) straight
- (2) Between any two distinct points we can draw straight line passing through them.
 - (a) zero
- (b) 1

- (c) 2
- (d) 3
- (3) If: $m (\angle A) + m (\angle B) = 180^{\circ}$, then $\angle A$ and $\angle B$ are
 - (a) equal in measure.

(b) complementary.

(c) supplementary.

- (d) adjacent.
- (4) If: $\overrightarrow{BA} \perp \overrightarrow{BC}$, then m ($\angle ABC$) =
 - (a) 40°
- (b) 90°
- (c) 180°
- (d) 360°
- (5) If: $\angle A$ supplements $\angle B$, $\angle A$ supplements $\angle C$, then $\angle B$ and $\angle C$ are
 - (a) equal in measure.

(b) complementary.

(c) supplementary.

- (d) adjacent.
- (6) If: $m (\angle X) = 15^{\circ}$, then the two angles whose measures are $2 m (\angle X)$, $4 m (\angle X)$ are
 - (a) complementary.

(b) supplementary.

(c) equal in measure.

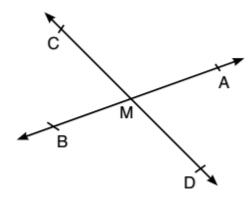
- (d) obtuse angles.
- (7) If: $m (\angle A) = 2 m (\angle B)$, $\angle A$ supplements $\angle B$, then $m (\angle B) = \cdots$
 - (a) 30°
- $(b) 60^{\circ}$
- (c) 120°
- (d) 90°

- (8) $\overline{AB} \cdots \overline{AB}$
 - (a) ∈
- (b)**∉**

- (c) C
- (d) ⊄
- (9) If: $m (\angle X) = 2 m (\angle Y)$ and $\angle Y$ is an obtuse angle, then $\angle X$ is
 - (a) acute.
- (b) right.
- (c) obtuse.
- (d) reflex.

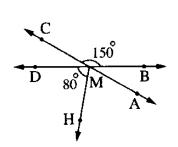
Sheet (3) Some Relations Between Angles (follow)

vertically opposite angles

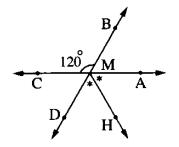


If two straight lines intersect, then the measures of each two vertically opposite angles are equal.

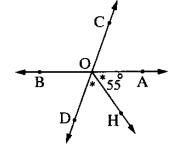
[1] In each figure, find the measure of the required angle:



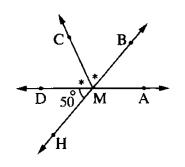
(1) m (\(\angle \) AMH) = \(\cdots \).



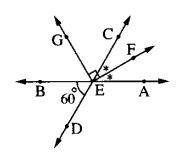
(2) m (∠ HMD) = ······°



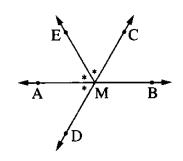
(3) m (\angle COB) = ···········



(4) m (∠ AMC) = ······°



(5) m (\angle GEB) = ·······°



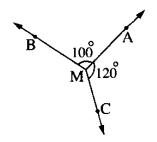
(6) m (\angle DMB) = ·······°



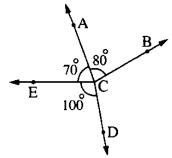
Accumulative angles at a point

The sum of the measures of the accumulative angles at a point is 360°

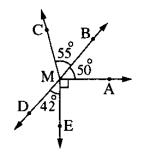
[2] In each figure, find the measure of the required angle:



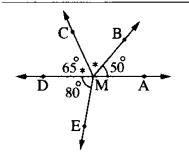
(7) m (\angle BMC) = ·······°



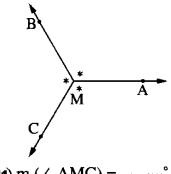
(8) m (∠ BCD) = ······°



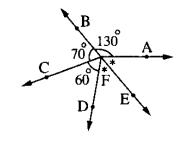
(9) m (∠ CMD) =·····°



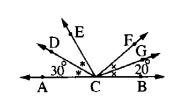
(10) m (∠ AME) = ······°



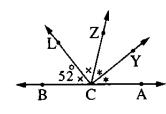
(11) m (∠ AMC) = ······°



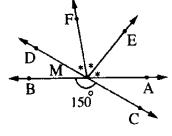
| (12) m (∠ EFD) = ······°



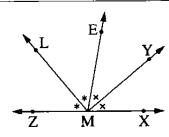
(13) m (∠ FCE) = ······°



(14) m (∠ YCA) = ······°



(15) m (∠ CMF) = ······°



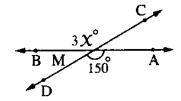
(16) m (∠ YML) = ······°

[3] Complete:

(2)	The sum of the measures of the accumulative angles at the point equals
-----	--

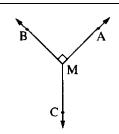


$$\overrightarrow{AB} \cap \overrightarrow{CD} = \{M\}$$
, then $X = \cdots$

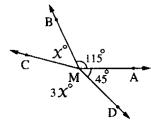


(4) In the opposite figure:

$\overrightarrow{MB} \perp \overrightarrow{MA}$ and \overrightarrow{MC}
bisects the reflexed angle AMB
then m (\angle AMC) = ·······°



(5) In the opposite figure:





It is the ray that divides the angle into two halves.

If \overrightarrow{BD} bisects $\angle ABC$

and m (\angle ABD) = 35°, then m (\angle ABC) =°



[4] Choose the correct answer:

- (1) The sum of the measures of the accumulative angles at the point equals angles.
 - (a) 2 right
- (b) 3 right
- (c) 4 right
- (d) 5 right
- The sum of measures of 4 accumulative angles at the point the sum of measures of 5 accumulative angles at the point.
 - (a) =
- (b) <
- (c) >

(d) ≠

- The two bisectors of two adjacent supplementary angles
 - (a) are perpendicular.

(b) are parallel.

(c) are coincident

(d) included an acute angle between them.

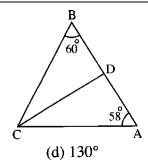
(4) In the opposite figure:

If ABC is a triangle in which \overrightarrow{CD}

bisects \angle ACB, m (\angle A) = 58°,

 $m (\angle B) = 60^{\circ}$

- , then m (\angle ADC) =
- (a) 62°
- (b) 89°
- (c) 91°



(5) In the opposite figure :

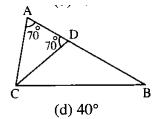
If \overrightarrow{CD} bisects \angle BCA, m (\angle A) = m (\angle ADC) = 70° ,

then m ($\angle B$) =

(a) 70°

(b) 30°

(c) 80°



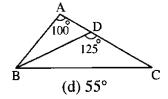
(6) In the opposite figure :

ABC is triangle, $D \subseteq \overline{AC}$ and \overline{BD} is a bisector of $\angle B$, what is the measure of $\angle C$?

(a) 25°

 $(b) 30^{\circ}$

 $(c) 45^{\circ}$



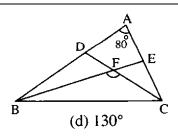
(7) In the opposite figure :

 $\underline{m} (\angle A) = 80^{\circ} , \overline{BE}$ is the bisector of $\angle B$, \overline{CD} is the bisector of $\angle C$ what is the measure of the shown angle BFC?

 $(a) 80^{\circ}$

(b) 100°

(c) 120°



[4] Answer the following:

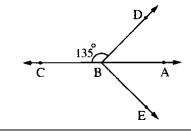
(1) In the opposite figure:

If $B \in \overrightarrow{AC}$, $m (\angle DBC) = 135^{\circ}$

and \overrightarrow{BA} bisects \angle DBE

Find each of:

 $m (\angle ABD) , m (\angle DBE) , m (\angle CBE)$



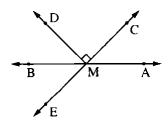
(2) In the opposite figure:

If $\overrightarrow{AB} \cap \overrightarrow{CE} = \{M\}$, $\overrightarrow{MD} \perp \overrightarrow{CE}$, and \overrightarrow{MB} bisects \angle DME

Find the measures of the following angles:

∠ BME ,∠ DME ,∠ AMC

and ∠ AME



(3) In the opposite figure:

 $m (\angle AMB) = 60^{\circ}, m (\angle AME) = 120^{\circ},$

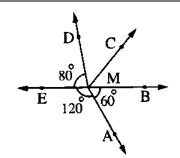
 $m (\angle EMD) = 80^{\circ}$

and \overline{MC} bisects \angle BMD

Find:

(1) m (\(\neq \cmp{CMD}\)

(2) m (∠ AMC)



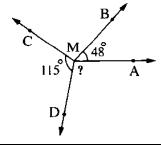
(4) In the opposite figure:

 $m (\angle BMC) = 2 m (\angle AMB)$,

 $m (\angle AMB) = 48^{\circ}$

and m (\angle DMC) = 115°

Find: $m (\angle AMD)$





- (1) Two line segments are congruent if they are equal in length. if AB = XY then $\overline{AB} \equiv \overline{XY}$.
- (2) Two angles are congruent if they are equal in measure. if $m(\angle A) = m(\angle B)$ then $\angle A = \angle B$.
- (3) Two polygons are congruent if each side and each angle in one of them are congruent to their corresponding elements in the other.
- (4) Two squares are congruent of the side length of one of them is congruent to the side length of the other.
- (5) Two rectangles are congruent if the dimensions of one of them are congruent to the dimensions of the other.



[1] Complete the following:

(1)	The two line segments are congruent if
(2)	The two angles are congruent if
(3)	The two polygons are congruent if there is a correspondence between their vertices such that each in the first polygon is congruent to its corresponding element in
(4)	The axis of symmetry of a polygon divides it into two polygons.
(5)	If $\overline{AB} = \overline{CD}$, then $AB = \dots$
(6)	If $\overline{AB} = \overline{XY}$, then $AB - XY = \dots$
(7)	If $\angle A \equiv \angle B$ and m ($\angle A$) = 50°, then m ($\angle B$) =°
(8)	If \angle A supplements \angle B and \angle A \equiv \angle B, then m (\angle B) =°
(9)	If $\angle A$ complements $\angle B$ and $\angle A \equiv \angle B$, then m ($\angle A$) =°
(10)	If C is the midpoint of \overline{AB} , then \overline{AC} \overline{BC}

If the polygon ABCD \equiv the polygon XYZL, then $\overline{DA} \equiv \cdots$ (11)

, m (\angle BCD) = m (\angle ······)

The two squares are congruent if are equal in length, while the two (12)rectangles are congruent if are equal.



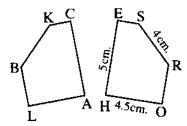
[2] Answer the following:

(1) In the opposite figure:

The two pentagons shown are congruent

Complete:

- (a) B corresponds to
- (2) The polygon BLACK is congruent to the polygon
- (3) KB = cm.
- (M) m $(\angle E)$ = m $(\angle \cdots)$
- $CA = \cdots cm$.
- (a) $m (\angle A) = m (\angle \cdots)$



(2) In the opposite figure:

> If $C \in \overrightarrow{BD}$, $m (\angle AFC) = 110^{\circ}$, BC = 5 cm. and the polygon ABCF ≡ the polygon EDCF

Complete the following:

- (A) AB =
- (2) AF = ·······
- (3) CD =
- ン。 J10 5 cm.

- (4) CF is side.

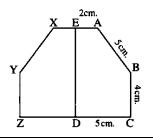
- (7) m (∠ FCD) = m (∠ ·······) (8) m (∠ EFC) = ·········° (10) m (∠ FCD) = ········° (11) m (∠ AFE) = ········°
- (9) BD = cm.
- (12) The axis of symmetry of the polygon ABDEF is
- (3)In the opposite figure:

If: $D \in CZ$ and the figure ABCDE = the figure XYZDE,

AE = 2 cm., BC = 4 cm. and AB = CD = 5 cm.



The perimeter of the figure ABCZYX = cm.

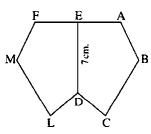


(4) In the opposite figure:

> If: $E \in \overrightarrow{AF}$, the perimeter of the figure ABCDE = 27 cm., DE = 7 cm.

and the polygon ABCDE = the polygon FMLDE

Find: The perimeter of the figure ABCDLMF = cm.

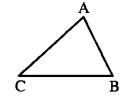


Sheet (5) Congruent triangles

We know that any triangle has three sides and three angles which are known as the six elements of the triangle.

For example:

 \triangle ABC has three sides which are : \overline{AB} , \overline{BC} and \overline{AC} and it has three angles which are : $\angle A$, $\angle B$ and $\angle C$



Therefore:

The two triangles are congruent if each element of the 6 elements of one of them is congruent to the corresponding element in the other triangle and vice versa.

• To test whether two triangles are congruent or not, you don't need to test all the three sides and the three angles.

The cases of congruence of two triangles

In the following, we will show the cases of congruence of two triangles. We will find that it is not necessary to prove congruence of the six elements of one of them to the corresponding elements of the other. But it is enough to prove congruence of three elements of the first to the corresponding elements of the other, one of them at least is a side, then the remained three elements in one of them are congruent to their corresponding elements in the other.

Cases of congruence of two triangles

Case (1)

Case (2)

*C*ase (3)

Case (4)

Two sides and the included angle

Two angles and one side

Three sides

Hypotenuse and one side in the right-angled triangle

S. A. S.

A. S. A.

S. S. S.

R. H. S.

Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle

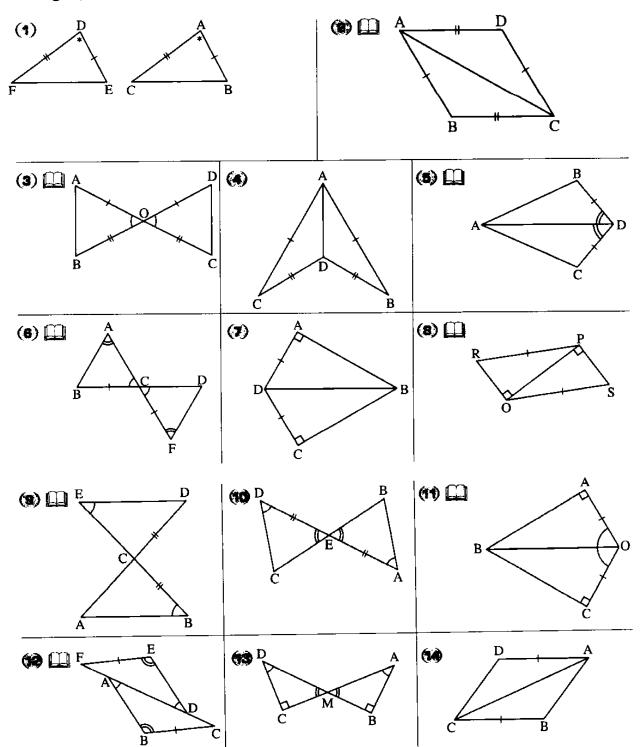
Two triangles are congruent if <u>each</u> side of one triangle is congruent to the corresponding side of the other triangle

Two right-angled
triangles are
congruent if the
hypotenuse and a side
of one triangle are
congruent to the
corresponding parts
of the other triangle

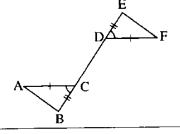
Remark

If each angle of one triangle is congruent to the corresponding angle of the other triangle, it is not necessary for the two triangles to be congruent.

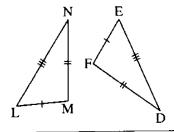
[1] In each of the following figures, show if the two triangles are congruent or not. If they are congruent, name the case of congruence. If they aren't congruent, give reason. (given that the similar signs denoted the congruency of the elements marked by these signs).



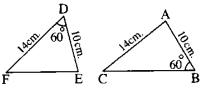
(15)



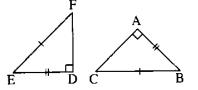
(16) 🛄



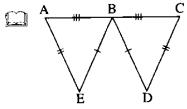
(17)



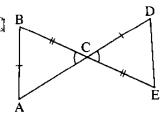
(18)



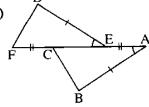
(19) 🛄



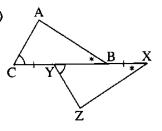
(20)



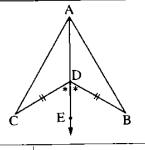
(21)



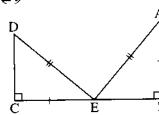
(22)



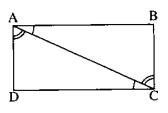
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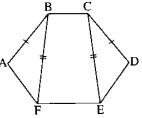
(24)



(25)



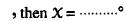
(26)





(1) In the opposite figure:

These triangles are congruent







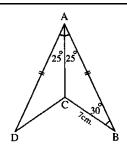
(2) In the opposite figure:

If: AB = AD, BC = 7 cm., $m (\angle BAC) = m (\angle DAC) = 25^{\circ}$

and m (\angle B) = 30°

Complete the following:

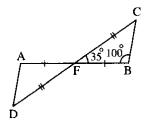
- (2) m (∠ D) = ······°
- (1) \triangle ACB \equiv \triangle (3) CD = ····· cm.
- (4) m (∠ ACD) = ······°



(3) In the opposite figure:

If:
$$\overline{CD} \cap \overline{BA} = \{F\}$$
, $FA = FB$, $CF = FD$, $m (\angle CFB) = 35^{\circ}$ and $m (\angle B) = 100^{\circ}$,

then m (\angle D) = ······°



(4) In the opposite figure:

If: BC = FD, $m(\angle A) = m(\angle E) = 95^{\circ}$,

 $m (\angle B) = 35^{\circ}$, $m (\angle D) = 50^{\circ}$ and FE = 7 cm.

Complete the following:

(1) m (\angle C) = ······°

- (**a**) m (∠ F) = ······°
- (**s**) ∆ ABC =

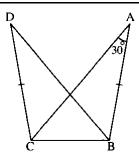
(4) AC ≡

- (5) $AB = \cdots cm$.
- (5) In the opposite figure:

If: AB = DC, AC = DB and m (\angle A) = 30°

Complete the following:

- (1) \triangle ABC \equiv \triangle
- (**2**) m (∠ D) = ······°
- (3) m (\angle DBC) = m (\angle ······)



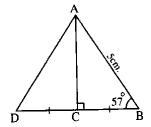
(6) In the opposite figure:

C is the midpoint of \overline{BD} , $\overline{AC} \perp \overline{BD}$,

AB = 5 cm. and $m (\angle B) = 57^{\circ}$

Find: (1) The length of \overline{AD}

(2) m $(\angle DAC)$

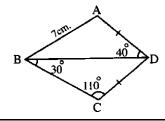


(7) In the opposite figure:

AD = DC, $m (\angle ADB) = 40^{\circ}$, $m (\angle DBC) = 30^{\circ}$,

m (\angle BCD) = 110° and AB = 7 cm.

Find: (1) The length of \overline{BC} (2) m (\angle BAD)



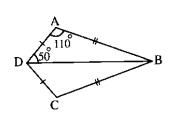
(8) In the opposite figure:

BA = BC, DA = DC,

 $m (\angle ADB) = 50^{\circ}$ and

 $m (\angle BAD) = 110^{\circ}$

Find: $m (\angle ABC)$

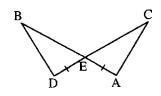


(9) In the opposite figure:

 $\overline{AB} \cap \overline{CD} = \{E\}$, AE = ED and $\angle A \equiv \angle D$

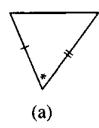
Is \triangle ACE \equiv \triangle DBE ? Why ?

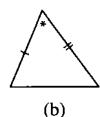
Prove that : CE = EB

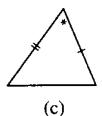


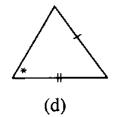
[3] Choose the correct answer:

(1) The following triangles are congruent except



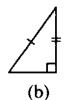




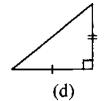


(2) The following triangles are congruent except

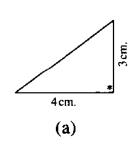


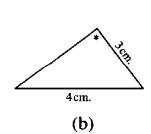


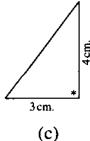


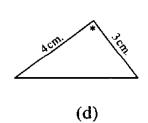


(3) The following triangles are congruent except

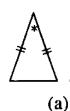




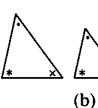


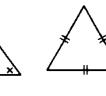


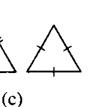
(4) The pair of congruent triangles of the following triangles is

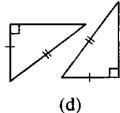












(5) In the opposite figure:

The necessary and enough condition which makes the two triangles ABC and XYZ be congruent is

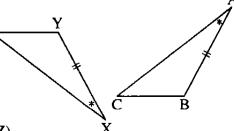




(c) m (\angle C) = m (\angle Z)

هذكرات جامزة للطباعة

(d) m ($\angle B$) = m ($\angle Z$)



[4] Complete the following:

- (1) If: \triangle ABC \equiv \triangle XYZ, m (\angle A) = 50° and m (\angle B) = 60°, then: m (\angle Z) =°
- (2) If: \triangle ABC \equiv \triangle LMN, m (\angle L) = 40° and m (\angle B) = 90°, then: m (\angle C) =°
- (3) If: \triangle ABC \equiv \triangle XYZ and m (\angle A) + m (\angle B) = 120°, then: m (\angle Z) =°
- (4) If: \triangle ABC \equiv \triangle DEF and m (\angle C) = 90°, then: m (\angle D) + m (\angle E) =°
- (5) If: \triangle ABC \equiv \triangle XYZ, the perimeter of \triangle ABC = 12 cm., XY = 4 cm. and YZ = 5 cm., then: AC =
- (6) Any two triangles are congruent if each is congruent to its corresponding side in the other triangle.
- (7) Any two triangles are congruent if two angles and in one of the triangles are congruent to their corresponding elements in the other.
- (8) The diagonal of the rectangle divides its surface into two triangles.
- (10) If: AB = LM, BC = MN and $m (\angle B) = m (\angle M)$, then the two triangles and will be congruent.

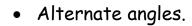




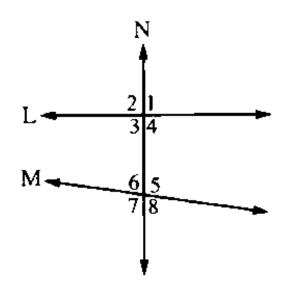
Angles Formed from two straight lines and a transversal:

If a straight line N cuts two straight lines L and M as shown in the opposite figure, then we get eight angles.

We can classify these angles into pairs of angles:

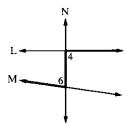


- Corresponding angles.
- Interior angles on the same side of the transversal.



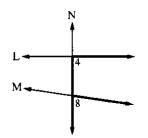
As follows

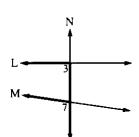
(1) Pairs of alternate angles:

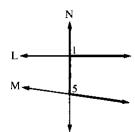


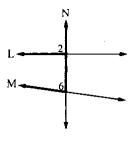
N N M 5

(2) Pairs of corresponding angles:

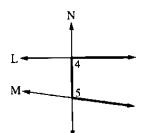


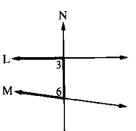






(3) Pairs of interior angles on the same side of the transversal





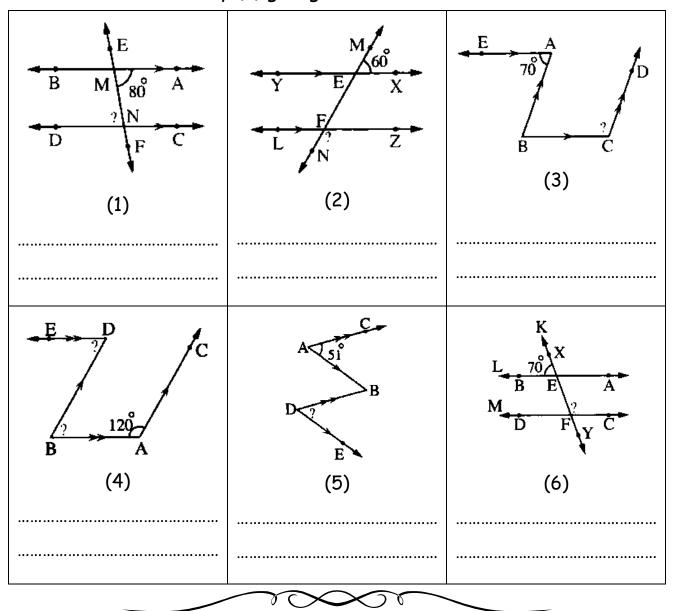
Relation between pairs of angles formed from two parallel straight lines and a transversal to them

If a straight line intersects two parallel lines, then:

- (1) Each two alternate angles are equal in measure.
- (2) Each two corresponding angles are equal in measure.
- (3) Each two interior angles in the same side of the transversal are supplementary.



In each of the following figures, find the measure of the angle which is marked by (?) giving reason:



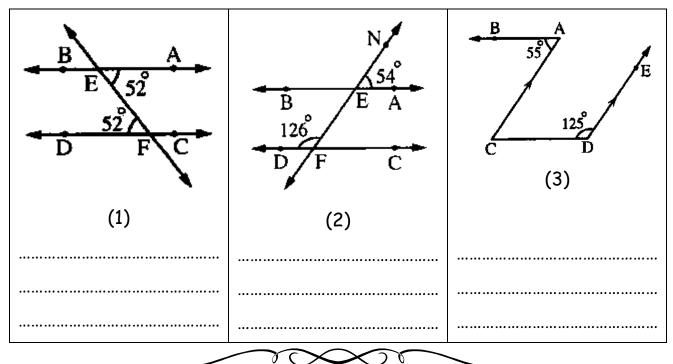
The condition of parallelism of two straight lines

The two straight lines are parallel if a third straight line intersects them (as a transversal) and one of the following cases satisfied:

- (1) Two alternate angles have the same measure.
- (2) Two corresponding angles have the same measure.
- (3) Two interior angles in the same side of the transversal are supplementary.



In each of the following figures, why is \overrightarrow{AB} // \overrightarrow{CD} ?



Geometric facts

- (1) The perpendicular to one of two parallel straight lines is perpendicular to the other.
- (2) If two straight lines are perpendicular to a third one, then the two straight lines are parallel.
- (3) If two straight lines are parallel to a third one, then the two straight lines are parallel.
- (4) If parallel straight lines divide a straight line into segments of equal lengths, then they divide any other line into segments of equal lengths.

H

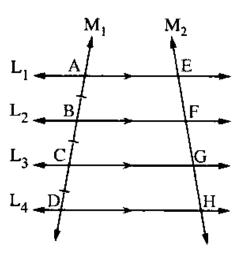
If L_1 // L_2 // L_3 // L_4 ,

and M_1 and M_2 are two transversal

in which:

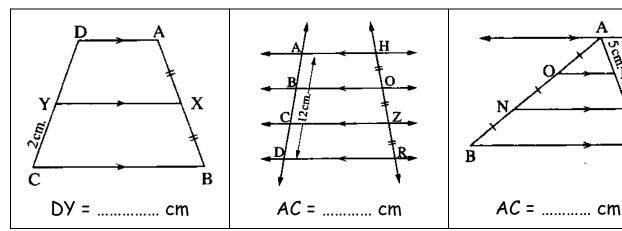
$$AB = BC = CD$$
,

then:





Complete using the given shown in the following figures:



[1] Choose the correct answer:

(1) In the opposite figure:

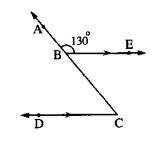
 $B \in \overline{AC}$, $\overrightarrow{BE} // \overrightarrow{CD}$ and m ($\angle ABE$) = 130°

Then m (\angle C) = ·······

- (a) 130°
- (b) 40°

(c) 50°

(d) 90°



(2) In the opposite figure:

 \overrightarrow{BE} bisects $\angle ABC$, $\overrightarrow{BA} / / \overrightarrow{CD}$ and

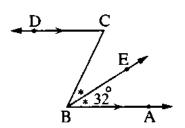
 $m (\angle ABE) = 32^{\circ}$, then $m (\angle C) = \cdots$

(a) 32°

(b) 64°

 $(c) 60^{\circ}$

(d) 80°



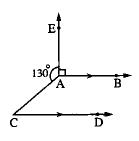
(3) In the opposite figure:

 $\overrightarrow{AB} // \overrightarrow{CD}$, m (\angle EAC) = 130°

and m (\angle EAB) = 90°, then m (\angle C) =

(a) 90°

- (b) 130°
- (c) 140°
- (d) 40°



(4) In the opposite figure:

 $\overrightarrow{AB} / / \overrightarrow{DE}$, m ($\angle D$) = 128°,

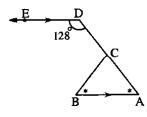
 $m (\angle A) = m (\angle B)$ and $C \subseteq \overline{AD}$, then $m (\angle B) = \cdots$

(a) 64°

(b) 128°

(c) 52°

(d) 26°

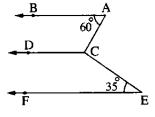


(5) In the opposite figure:

 \overrightarrow{AB} // \overrightarrow{CD} , \overrightarrow{AB} // \overrightarrow{EF} , m ($\angle A$) = 60° and

 $m (\angle E) = 35^{\circ}$, then $m (\angle ACE) = \cdots$

- (a) 60°
- (b) 35°
- (c) 95°
- (d) 85°



(6) In the opposite figure:

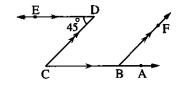
 $m (\angle D) = 45^{\circ} , \overrightarrow{DE} // \overrightarrow{CA}$ and

 $\overrightarrow{CD} / / \overrightarrow{BF}$, then m ($\angle ABF$) =

 $(a) 45^{\circ}$

- (b) 90°
- (c) 135°

(d) 40°



(7) In the opposite figure:

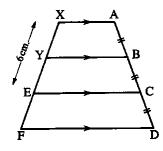
 $\overline{AX} // \overline{BY} // \overline{CE} // \overline{DF}$,

AB = BC = CD

and XE = 6 cm.

, then the length of $\overline{YF} = \cdots$

- (a) 3 cm.
- (b) 6 cm.
- (c) 12 cm.
- (d) 9 cm.



(8) In the opposite figure:

 $\overrightarrow{AB} / \overrightarrow{CF} / \overrightarrow{DE}$,

 $m (\angle A) = 120^{\circ} \text{ and } m (\angle D) = 85^{\circ}$,

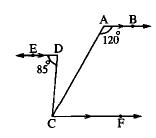
then m (\angle ACD) = ·······

(a) 60°

 $(b) 85^{\circ}$

(c) 25°

(d) 120°



(9) In the opposite figure:

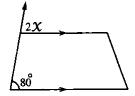
What is the value of X?

(a) 40°

(b) 60°

(c) 80°

(d) 100°

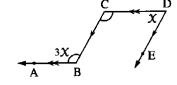


(10) In the opposite figure:

 $\overrightarrow{CD} / / \overrightarrow{BA}, \overrightarrow{DE} / / \overrightarrow{CB}$

- then : $x = \cdots$
- $(a) 60^{\circ}$

- (b) 45°
- (c) 120°
- (d) 90°



[2] Complete:

- (1) The straight line which is perpendicular to one of two parallel straight lines is to the other straight line in the plane.
- (2) If two straight lines are parallel to a third straight line, then they are
- (3) If a straight line cuts two parallel straight lines, then each two alternate angles are
- (4) If a straight line cuts two parallel straight lines, then each two corresponding angles are
- (5) If a straight line cuts two parallel straight lines, then each two interior angles in the same side of the transversal are
- (6) If a straight line cuts two straight lines and there are two corresponding angles having the same measure, then the two straight lines are
- (7) If a straight line cuts two straight lines and there are two alternate angles having the same measure, then the two straight lines are
- (8) If a straight line cuts two straight lines and there are two interior angles in the same side of the transversal are supplementary, then the two straight lines are
- (9) If a straight line cuts several parallel lines and the intercepted parts of this transversal between these parallel straight lines are equal in length, then the intercepted parts for any transversal are



[3] Answer the following:

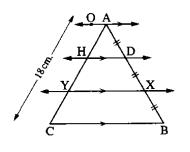
(1) In the opposite figure:

$$\overrightarrow{AO}$$
 // \overrightarrow{HD} // \overrightarrow{YX} // \overrightarrow{CB}

$$, AD = DX = XB$$

and
$$AC = 18 \text{ cm}$$
.

Find the length of \overline{AY}

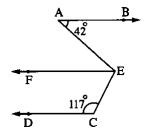


(2) In the opposite figure:

$$\overrightarrow{AB} / / \overrightarrow{CD}, \overrightarrow{EF} / / \overrightarrow{CD}$$

, m (
$$\angle$$
 A) = 42° and m (\angle C) = 117°

Determine: m (∠ AEC)

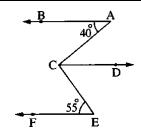


(3) In the opposite figure:

$$m (\angle A) = 40^{\circ}$$
, $m (\angle E) = 55^{\circ}$

 \overrightarrow{AB} // \overrightarrow{EF} and \overrightarrow{AB} // \overrightarrow{CD}

Find: $m (\angle ACE)$

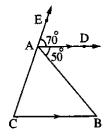


(4) In the opposite figure:

$$\overrightarrow{AD} / \overrightarrow{BC}, \overrightarrow{E} \in \overrightarrow{CA},$$

m (\angle DAE) = 70° and m (\angle DAB) = 50°

Find the measures of the triangle ABC



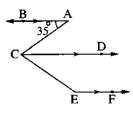
(5) In the opposite figure:

$$\overrightarrow{AB} / \overrightarrow{CD} / \overrightarrow{EF}$$
, m ($\angle A$) = 35° and

CD bisects ∠ ACE

Find: (1) m $(\angle DCE)$

(2) m (∠ CEF)

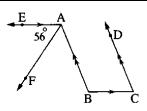


(6) In the opposite figure:

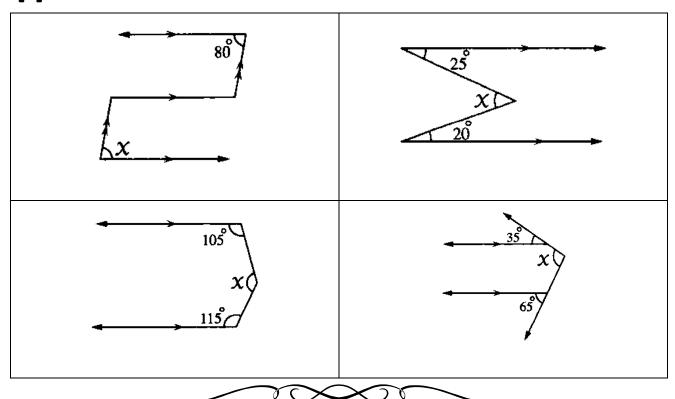
$$\overrightarrow{AE} / \overrightarrow{CB}, \overrightarrow{BA} / \overrightarrow{CD},$$

 \overrightarrow{AF} bisects \angle BAE and m (\angle EAF) = 56°

Find: $m (\angle C)$



[4] Find the value of X:



Sheet (7) Geometric constructions

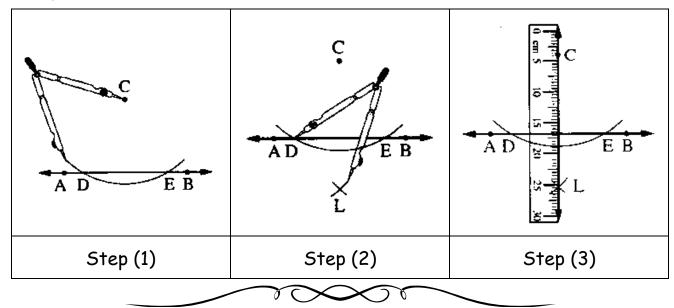
First: Constructing a perpendicular from a point outside a straight line:

If \overrightarrow{AB} is a given straight line, $C \notin \overrightarrow{AB}$ as shown in fig. (1)

The required is constructing the perpendicular to \overrightarrow{AB} from C



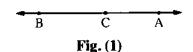
Steps:



Second: Constructing a perpendicular from a point on a straight line:

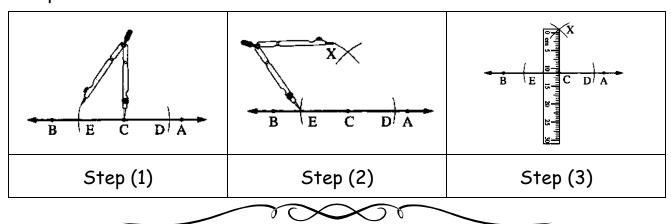
If: AB is a given straight line.

 $C \in \overrightarrow{AB}$ as shown in fig. (1)



The required is drawing a perpendicular to \overrightarrow{AB} from the point C

Steps:



The axis of symmetry of a line segment

It is the straight line perpendicular to it from its midpoint.



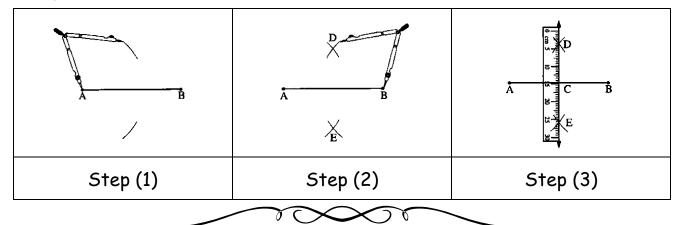
Third: Bisecting a given line segment:

If \overline{AB} is a given line segment as shown in fig. (1)

A Fig. (1)

The required is constructing the symmetry axis of the line segment \overline{AB} (The perpendicular to \overline{AB} from its midpoint).

Steps:

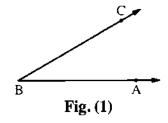


Fourth: Bisecting a given angle:

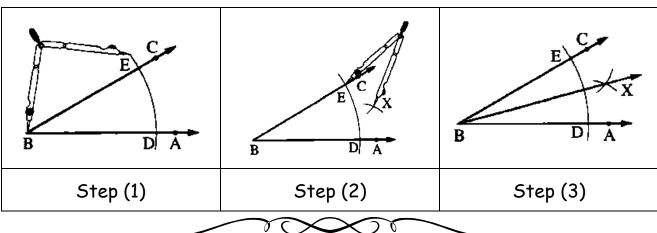
If \angle ABC is a given angle as shown in fig. (1)

The required is constructing the bisector of \angle ABC

"Using the compasses and the ruler"



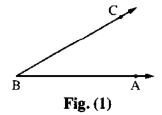
Steps:



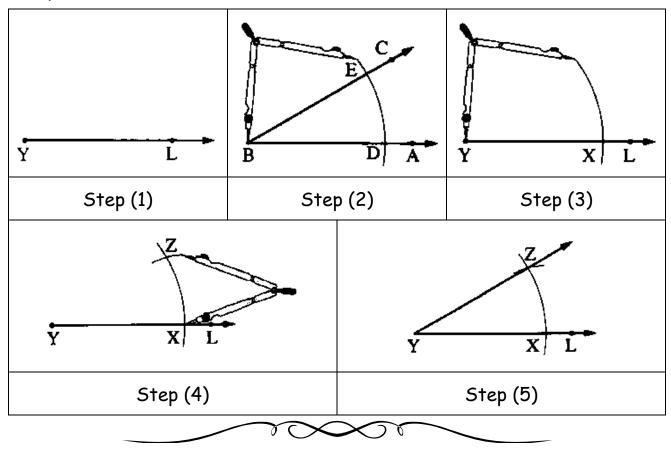
Fifth: Constructing an angle to be congruent to a given angle:

 \angle ABC is a given angle as shown in fig. (1) The required is drawing \angle XYZ such that \angle XYZ is congruent to \angle ABC

i.e.: $m (\angle XYZ) = m (\angle ABC)$



Steps:



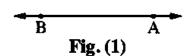
Using the ruler and the compasses, draw \triangle ABC in which AB = AC = 5 cm., BC = 6 cm., then draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$ Then find by measuring the length of \overline{AD} (Don't remove the arcs)

Sixth: Drawing a straight line from a given point parallel to given straight line:

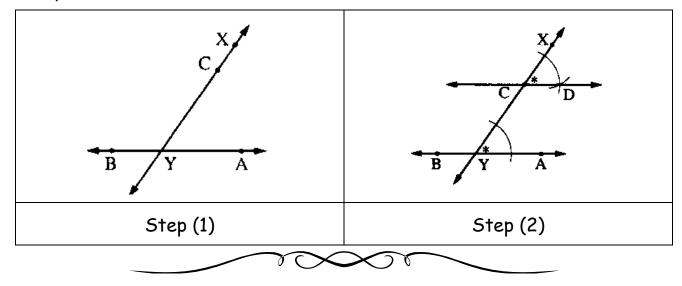
 \overrightarrow{AB} is a given straight line and $C \notin \overrightarrow{AB}$ as shown in fig. (1)

C.

Required: The drawing a straight line passing through the point C parallel to \overrightarrow{AB}



Steps:



Using the ruler and the compasses, draw the line segment \overline{BC} with length 7 cm., then draw the straight line L as an axis of symmetry of it. (Don't remove the arcs)

Draw an angle whose vertex is A and its measure is 130°, use a ruler and a compasses to divide the angle A into 4 equal angles in measure. (Don't remove the arcs)



Using the geometric instruments, draw an angle of measure 120° and bisect it (Don't remove the arcs).



Using the geometric tools, draw an angle of measure 75° and bisect it (Don't remove the arcs).